DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

2006

6 (a) Differentiate $\sqrt{x}(x+2)$ with respect to x

(b) The equation of a curve is $y = 3x^4 - 2x^3 - 9x^2 + 8$.

(i) Show that the curve has a local maximum at the point (0, 8).

(ii) Find the coordinates of the two local minimum points on the curve.

(iii) Draw a sketch of the curve.

(c) Prove by induction that $\frac{d}{dx}(x^n) = nx^{n-1}$, $n \geq 1$, $n \in \mathbf{N}$.

7 (a) Taking $x_1 = 2$ as the first approximation to the real root of the equation $x^3 + x - 9 = 0$, use the Newton-Raphson method to find x_2, the second approximation.

(b) The parametric equations of a curve are:

$$x = 3\cos\theta - \cos^3\theta$$
$$y = 3\sin\theta - \sin^3\theta, \text{ where } 0 < \theta < \tfrac{\pi}{2}.$$

(i) Find $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$.

(ii) Hence show that $\frac{dy}{dx} = \frac{-1}{\tan^3\theta}$.

(c) Given $y = \ln\left(\frac{3+x}{\sqrt{9-x^2}}\right)$, find $\frac{dy}{dx}$ and express it in the form $\frac{a}{b-x^n}$.

ANSWERS

6 (a) $\frac{3}{2}\sqrt{x} + \frac{1}{\sqrt{x}}$

6 (b) (ii) $(-1, 4)$, $(\tfrac{3}{2}, -\tfrac{61}{16})$

7 (a) $\tfrac{25}{13}$

7 (b) (i) $\frac{dy}{d\theta} = 3\cos^3\theta$, $\frac{dx}{d\theta} = -3\sin^3\theta$

7 (c) $\frac{3}{9-x^2}$