DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

2004

6 (a) Differentiate $\frac{1}{2+5x}$ with respect to x.

(b) (i) Given $y = \tan^{-1} x$, find the value of $\frac{dy}{dx}$ at $x = \sqrt{2}$.

(ii) Differentiate, from first principles, $\cos x$ with respect to x.

(c) Let $f(x) = x^3 + 6x^2 + 15x + 36,\ x \in \mathbf{R}$.

(i) Show that $f'(x)$ can be written in the form $3[(x+a)^2 + b],\ a, b \in \mathbf{R}$, where $f'(x)$ is the first derivative of $f(x)$.

(ii) Hence show that $f(x) = 0$ has only one real root.

7 (a) An object's distance from a fixed point is given by $s = 12 + 24t - 3t^2$, where s is in metres and t is in seconds. Find the speed of the object when $t = 3$ seconds.

(b) The parametric equations of a curve are:

$$x = 2\theta - \sin 2\theta$$
$$y = 1 - \cos 2\theta, \text{ where } 0 < \theta < \pi.$$

(i) Find $\frac{dy}{dx}$.

(ii) Show that the tangent to the curve at $\theta = \frac{\pi}{6}$ is perpendicular to the tangent at $\theta = \frac{2\pi}{3}$.

(c) Given that $x = \frac{e^{2y} - 1}{e^{2y} + 1}$,

(i) show that $e^{2y} = \frac{1+x}{1-x}$

(ii) show that $\frac{dy}{dx}$ can be expressed in the form $\frac{p}{1-x^p}$, $p, q \in \mathbf{N}$.

ANSWERS

6 (a) $\frac{dy}{dx} = -\frac{5}{(2+5x)^2}$

6 (b) (i) $\left(\frac{dy}{dx}\right)_{x=\sqrt{2}} = \frac{1}{3}$

6 (c) (i) $f'(x) = 3[(x+2)^2+1]$ (ii) $f'(x) = 3[(x+2)^2+1] > 0$

7 (a) $\left(\frac{ds}{dt}\right)_{t=3} = 6\,\mathrm{m\,s^{-1}}$

7 (b) (i) $\frac{dy}{dx} = \frac{\sin 2\theta}{1-\cos 2\theta}$

7 (c) (ii) $\frac{dy}{dx} = \frac{1}{1-x^2}$