DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

2003

6 (a) Differentiate $\sqrt{1+4x}$ with respect to x.

(b) Show that the equation $x^3 - 4x - 2 = 0$ has a root between 2 and 3.
Taking $x_1 = 2$ as the first approximation to this root, use the Newton-Raphson method to find x_3, the third approximation. Give your answer correct to two decimal places.

(c) The function $f(x) = \dfrac{1}{1-x}$ is defined for $x \in \mathbf{R} \setminus \{1\}$.

(i) Prove that the graph of f has no turning points and no points of inflection.

(ii) Write down the reason that justifies the statement: "f is increasing at every value of $x \in \mathbf{R} \setminus \{1\}$."

(iii) Given that $y = x + k$ is a tangent to the graph of f where k is a real number, find the two possible values of k.

7 (a) Differentiate each of the following with respect to x:

(i) $\cos^4 x$ (ii) $\sin^{-1}(\frac{x}{5})$.

(b) (i) The parametric equations of a curve are:

$$x = \cos t + t \sin t$$
$$y = \sin t - t \cos t \quad \text{where } 0 < t < \tfrac{\pi}{2}.$$

Find $\dfrac{dy}{dx}$ and write your answer in its simplest form.

(ii) Given that $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{6}$, find the value of $\dfrac{dy}{dx}$ at the point $(2, -3)$.

(c) (i) Given that $y = \ln \dfrac{1+x^2}{1-x^2}$ for $0 < x < 1$, find $\dfrac{dy}{dx}$ and write your answer in the form $\dfrac{kx}{1-x^k}$ where $k \in \mathbf{N}$.

(ii) Given that $f(\theta) = \sin(\theta + \pi)\cos(\theta - \pi)$, find the derivative of $f(\theta)$ and express it in the form $\cos n\theta$ where $n \in \mathbf{Z}$.

ANSWERS

6 (a) $\frac{dy}{dx} = \frac{2}{\sqrt{1+4x}}$

6 (b) $x_3 = 2 \cdot 22$

6 (c) (iii) $k = -3, 1$

7 (a) (i) $\frac{dy}{dx} = -4\cos^3 x \sin x$ (ii) $\frac{dy}{dx} = \frac{1}{\sqrt{25-x^2}}$

7 (b) (i) $\frac{dy}{dx} = \tan t$ (ii) $\left(\frac{dy}{dx}\right)_{(2,\,-3)} = -\frac{9}{4}$

7 (c) (i) $\frac{dy}{dx} = \frac{4x}{1-x^4}$ (ii) $f'(\theta) = \cos 2\theta$