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DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

2002

6 (a) Differentiate $(x^4+1)^5$ with respect to x.

(b) (i) Prove, from first principles, the addition rule:

if $f(x)=u(x)+v(x)$ then $\frac{df}{dx}=\frac{du}{dx}+\frac{dv}{dx}$.

(ii) Given $y=2x-\sin 2x$, find $\frac{dy}{dx}$. Give your answer in the form $k\sin^2 x$, where $k\in\mathbf{Z}$.

(c) The function $f(x)=ax^3+bx^2+cx+d$ has a maximum point at (0, 4) and a point of inflection at (1, 0). Find the values of a, b, c and d.

7 (a) Find the slope of the tangent to the curve $9x^2+4y^2=40$ at the point (2, 1).

(b) (i) Given that $y=\sin^{-1}10x$, evaluate $\frac{dy}{dx}$ when $x=\frac{1}{20}$.

(ii) The parametric equations of a curve are $x=\ln(1+t^2)$ and $y=\ln 2t$, where $t\in\mathbf{R}$, $t>0$. Find the value of $\frac{dy}{dx}$ when $t=\sqrt{5}$.

(c) Let $f(x)=\frac{e^x+e^{-x}}{2}$.

(i) Show that $f''(x)=f(x)$, where $f''(x)$ is the second derivative of $f(x)$.

(ii) Show that $\frac{f'(2x)}{f'(x)}=2f(x)$ when $x\neq 0$ and where $f'(x)$ is the first derivative of $f(x)$.

ANSWERS

6 (a) $\frac{dy}{dx}=20x^3(x^4+1)^4$

6 (b) (ii) $\frac{dy}{dx}=4\sin^2 x$

6 (c) $a=2,\ b=-6,\ c=0,\ d=4;\ f(x)=2x^3-6x^2+4$

7 (a) $\left(\frac{dy}{dx}\right)_{(2,1)}=-\frac{9}{2}$

7 (b) (i) $\left(\frac{dy}{dx}\right)_{x=\frac{1}{20}}=\frac{20}{\sqrt{3}}$ (ii) $\left(\frac{dy}{dx}\right)_{t=\sqrt{5}}=\frac{3}{5}$