Differentiation & Applications (Q 6 & 7, Paper 1)

2001

6 (a) Differentiate $\frac{x}{1+x^2}$ with respect to x.

(b) (i) Given that $y=\sqrt{x}$, what is $\frac{dy}{dx}$?

(ii) Now, find from first principles the derivative of $\sqrt{x}$ with respect to x.

(c) Let $x=t^2e^t$ and $y=t+2\ln t$ for $t>0$.

(i) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$ in terms of t.

(ii) Hence, show that $\frac{dy}{dx}=\frac{1}{x}$.

7 (a) Taking $x_1=1$ as the first approximation to the real root of the equation $x^3+x^2-1=0$, use the Newton-Rhapson method to find x_2, the second approximation.

(b) (i) Differentiate $\tan^{-1}7x$ with respect to x.

(ii) Given that $y=\sin x\cos x$, find $\frac{dy}{dx}$ and express it in the form $\cos nx$ where $n\in\mathbf{Z}$.

(c) Let $g(x)=x^2+\frac{a}{x^2}$ where a is a real number and $x\in\mathbf{R}$, $x\neq 0$. Given that $g(x)$ has a turning point at $x = 2$,

(i) find the value of a

(ii) prove that $g(x)$ has no local maximum points.

Answers

6 (a) $\frac{dy}{dx}=\frac{1-x^2}{(1+x^2)^2}$

6 (b) (i) $\frac{dy}{dx}=\frac{1}{2\sqrt{x}}$

6 (c) (i) $\frac{dx}{dt}=te^t(t+2)$, $\frac{dy}{dt}=1+\frac{2}{t}$

7 (a) $x_2=\frac{4}{5}=0\cdot 8$

7 (b) (i) $\frac{dy}{dx}=\frac{7}{1+49x^2}$ (ii) $\frac{dy}{dx}=\cos 2x$

7 (c) (i) $a = 16$