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Differentiation & Applications (Q 6 & 7, Paper 1)

2000

6 (a) Differentiate with respect to x

(i) $(1+5x)^3$ (ii) $\frac{7x}{x-3}$, $x \neq 3$.

(b) (i) Prove, from first principles, the product rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

where $u = u(x)$ and $v = v(x)$.

(ii) Given $y = \sin^{-1}(2x-1)$, find $\frac{dy}{dx}$ and calculate its value at $x = \frac{1}{2}$.

(c) $f(x) = \frac{1}{x+1}$ where $x \in \mathbf{R}, x \neq -1$.

(i) Find the equations of the asymptotes of the graph of $f(x)$.

(ii) Prove that the graph of $f(x)$ has no turning points or points of inflection.

(iii) If the tangents to the curve at $x = x_1$ and $x = x_2$ are parallel and if $x_1 \neq x_2$, show that

$$x_1 + x_2 + 2 = 0.$$

7 (a) Find the slope of the tangent to the curve $x^2 - xy + y^2 = 1$ at the point (1, 0).

(b) The parametric equations of a curve are $x = \cos^3 t$ and $y = \sin^3 t$, $0 \leq t \leq \frac{\pi}{2}$.

(i) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$ in terms of t.

(ii) Hence, find integers a and b such that $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \frac{a}{b}(\sin t)^2$.

(c) $f(x) = \frac{\ln x}{x}$ where $x > 0$.

(i) Show that the maximum of $f(x)$ occurs at the point $(e, \frac{1}{e})$.

(ii) Hence, show that $x^e \leq e^x$ for all $x > 0$.

Answers

6 (a) (i) $15(1+5x)^2$ (ii) $-\frac{21}{(x-3)^2}$

6 (b) (ii) 2

6 (c) (i) $x = -1$, $y = 0$

7 (a) 2

7 (b) (i) $\frac{dx}{dt} = -3\cos^2 t \sin t$, $\frac{dy}{dt} = 3\sin^2 t \cos t$

(ii) $a = 9$, $b = 4$