Differentiation & Applications (Q 6 & 7, Paper 1)

1999

6 (a) Differentiate

$(3-4x)^5$ with respect to x.

(b) Find from first principles the derivative of $\sin x$ with respect to x.

(c) Let $f(x) = xe^{-ax}$, $x \in \mathbf{R}$, a constant and $a > 0$.
Show that $f(x)$ has a local maximum and express the coordinates of this local maximum point in terms of a.
Find, in terms of a, the coordinates of the point at which the second derivative of $f(x)$ is zero.

7 (a) Find the derivative of $\sqrt{x^2+1}$.

(b) (i) Let $x = t - \sin t \cos t$ and $y = 4\cos t$, $0 < t < \frac{\pi}{2}$.

Show that $\frac{dy}{dx} = -\frac{2}{\sin t}$.

(ii) Find the slope of the tangent to the curve
$x^2 - y^2 - x = 1$ at the point (2, 1).

(c) Let $f(x) = x^3 + kx^2 - 4$, $x \in \mathbf{R}$ and $k > 0$.
Show that the coordinates of the local minimum and local maximum of $f(x)$ are

$(0, -4)$ and $\left(-\frac{2k}{3}, \frac{4k^3 - 108}{27}\right)$, respectively.

Find
(i) the range of values of k for which $f(x) = 0$ has three real roots
(ii) the value of k for which $f(x) = 0$ has three roots, two of which are equal.

Answers

6 (a) $-20(3-4x)^4$

6 (c) $\left(\frac{1}{a}, \frac{1}{ae}\right), \left(\frac{2}{a}, \frac{2}{ae^2}\right)$

7 (a) $\frac{x}{\sqrt{x^2+1}}$

7 (b) (ii) $\frac{3}{2}$

7 (c) (i) $k > 3$ (ii) $k = 3$