DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

1998

6 (a) Differentiate (i) $(1+3x)^2$ (ii) $3e^{4x+1}$.

(b) Find the value of the constant k if $y = kx^2$ is a solution of the equation

$$x\frac{dy}{dx}+\frac{1}{2}\left(\frac{dy}{dx}\right)^2+y=0,$$

where $x \in \mathbf{R}$ and $k \neq 0$.

(c) Given that $f(x)=\dfrac{x}{x+2}, x \in \mathbf{R}$ and $x \neq 2$,

find the equations of the asymptotes of the graph of $f(x)$.
Prove that the graph of $f(x)$ has no turning points or points of inflection.
Find the range of values of x for which $f'(x) \leq 1$, where $f'(x)$ is the derivative of $f(x)$.

7 (a) Let $\theta = 5t^3 - 2t^2$,
where t is in seconds and θ is in radians.
Find the rate of change of θ when $t = 2$ seconds.

(b) The parametric equations of a curve are

$$x=\frac{1+\sin t}{\cos t},\ y=\frac{1+\cos t}{\sin t},\ 0<t<\tfrac{\pi}{2}.$$

Find $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$.

Find the slope of the tangent to the curve at the point where $t=\tan^{-1}(\tfrac{3}{4})$.

(c) Let $f(x)=x^3-kx^2+8,\ k \in \mathbf{R}$ and $k>0$.

Show that the coordinates of the local minimum point of $f(x)$ are $(\tfrac{2k}{3}, 8-\tfrac{4k^3}{27})$.
Taking $x_1 = 3$ as the first approximation of one of the roots of $f(x) = 0$, the Newton-Raphson method gives the second approximation as $x_2=\tfrac{10}{3}$.
Find the value of k.

ANSWERS

6 (a) (i) $6(1+3x)$ (ii) $12e^{4x+1}$

(b) $k=-\tfrac{3}{2}$

(c) $x=-2, y=1;\ x\leq -2-\sqrt{2},\ x\geq -2+\sqrt{2}$

7 (a) 52 rads/s

(b) $\dfrac{dx}{dt}=\dfrac{1+\sin t}{\cos^2 t},\ \dfrac{dy}{dt}=\dfrac{-1-\cos t}{\sin^2 t},\ -2$

(c) $k = 4$