Differentiation & Applications (Q 6 & 7, Paper 1)

1997

6 (a) Differentiate

(i) $x^3 + 2\sqrt{x}$ (ii) $(x+2)\ln x$.

(b) (i) Find from first principles the derivative of x^3 with respect to x.

(ii) Let $f(x) = \sin^4 x + \cos^4 x$.

Find the derivative of $f(x)$ and express it in the form $k \sin px$, where $k, p \in \mathbf{Z}$.

(c) If $\sin y = \frac{1}{2}(1 - x^2)$ for $-\sqrt{3} < x < \sqrt{3}$,
calculate the value of a and the value of b when

$$\left(\frac{dy}{dx}\right)^2 = \frac{a}{3 - x^2} - \frac{b}{1 + x^2},\ a, b \in \mathbf{N}_0.$$

7 (a) Take $x_1 = 3$ as the first approximation of a real root of the equation
$x^3 - 6x^2 + 24 = 0$.
Find, using the Newton-Raphson method, x_2, the second approximation and write your answer as a fraction.

(b) (i) Find the equation of the tangent to the curve

$2x^2 - 3y^2 = 6$

at the point $(-3, -2)$.

(ii) If $x = \dfrac{1 - t^2}{1 + t^2}$ and $y = \dfrac{2t}{1 + t^2}$, find, as a fraction, the value of $\dfrac{dy}{dx}$ when $t = \frac{3}{4}$.

(c) Let $y = x - 1 + \dfrac{1}{x - 1}$, $x \in \mathbf{R}$, $x \neq 1$.

(i) Find the values of x for which $\dfrac{dy}{dx} = 0$.

(ii) For x real, show that y cannot have a real value between -2 and $+2$.

Answers

6 (a) (i) $3x^2 + \dfrac{1}{\sqrt{x}}$ (ii) $\dfrac{x+2}{x} + \ln x$

(b) (ii) $-\sin 4x$

(c) $a = 3, b = 1$

7 (a) $\frac{8}{3}$

(b) $x - y + 1 = 0$

(c) (i) $x = 0, 2$