DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

1996

6 (a) Differentiate

(i) $\dfrac{2x}{x+1}$ (ii) $4e^{2x+1}$

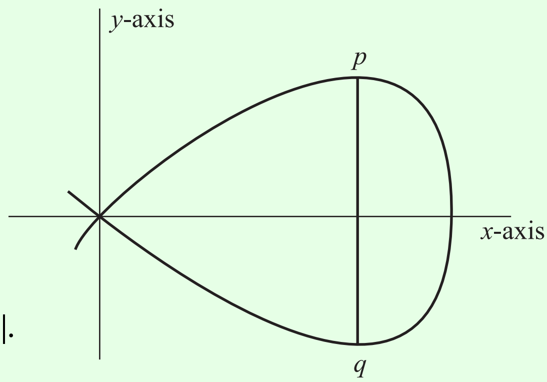
(b) (i) Find $\dfrac{dy}{dx}$ if $y = \ln\sqrt{x^2+1}$.

(ii) Take $x_1 = 1$ as the first approximation of a real root of the equation $x^3 - 2 = 0$. Find, using the Newton-Raphson method, x_2 and x_3 the second and third approximations. Write your answers as fractions.

(c) (i) $x = a(\theta + \sin\theta)$; $y = a(1-\cos\theta)$ where a is a constant.
Show

$$1+\left(\frac{dy}{dx}\right)^2 = \sec^2\left(\tfrac{\theta}{2}\right).$$

(ii) $[pq]$ is a chord of the loop of the curve $y^2 = x^2(6-x)$ so that the chord is parallel to the y-axis.
Calculate the maximum value of $|pq|$.



7 (a) Find form first principles the derivative of x^2 with respect to x.

(b) The function f is defined

$$f : x \to (x-4)\{(x-3)^2+4\}.$$

Find

(i) $f(3)$

(ii) the derivative with respect to x of the function at $x = 3$.

(iii) the equation of the tangent at $(3, f(3))$.

Show that the tangent and the graph of $x \to f(x)$ will both intersect the x-axis at the same point.

(c) (i) Given $\tan y = x$, show $\dfrac{dy}{dx} = \dfrac{1}{1+\tan^2 y}$ and hence, find $\dfrac{d}{dx}\tan^{-1} x$.

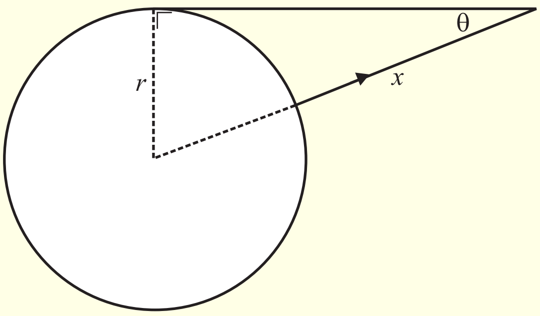
(ii) An astronaut is at a height x km above the earth, as shown.

He moves vertically away from the earth's surface at a velocity $\dfrac{dx}{dt}$ of $\dfrac{r}{5}$ km/h where r is the length of the earth's radius.

He observes the angle θ as shown.

Express x in terms of r and θ.

Hence find $\dfrac{d\theta}{dt}$ when $x = r$.



ANSWERS

6 (a) (i) $\dfrac{2}{(x+1)^2}$ (ii) $8e^{2x+1}$

(b) (i) $\dfrac{x}{x^2+1}$ (ii) $x_2 = \frac{4}{3}, x_3 = \frac{91}{72}$

(c) (ii) $8\sqrt{2}$

7 (b) (i) -4 (ii) 4 (iii) $y = 4x - 16$

(c) (i) $\dfrac{1}{1+x^2}$ (ii) $-\dfrac{1}{10\sqrt{3}}$