Complex Numbers & Matrices (Q 3, Paper 1)

Lesson No. 3: De Moivre's Theorem

2006

3 (c) (i) Express $-8-8\sqrt{3}i$ in the form $r(\cos\theta + i\sin\theta)$.

(ii) Hence find $(-8-8\sqrt{3}i)^3$.

(iii) Find the four complex number z such that $z^4 = -8-8\sqrt{3}i$. Give your answers in the form $a+bi$, with a and b fully evaluated.

2005

3 (c) (i) $z = \cos\theta + i\sin\theta$. Use De Moivre's theorem to show that $z^n + \frac{1}{z^n} = 2\cos n\theta$, for $n \in \mathbf{N}$.

(ii) Expand $\left(z+\frac{1}{z}\right)^4$ and hence express $\cos^4\theta$ in terms of $\cos 4\theta$ and $\cos 2\theta$.

2004

3 (b) (i) $z_1 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$ and $z_2 = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$. Evaluate $z_1 z_2$, giving your answer in the form $x+iy$.

2003

3 (c) $1, \omega, \omega^2$ are the three roots of the equation $z^3 - 1 = 0$.

(i) Prove that $1+\omega+\omega^2 = 0$.

(ii) Hence, find the value of $(1-\omega-\omega^2)^5$.

2002

3 (a) Express $-1+\sqrt{3}i$ in the form $r(\cos\theta + i\sin\theta)$, where $i^2 = -1$.

Answers

2006 3 (c) (i) $16(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3})$ (ii) 4096 (iii) $1+\sqrt{3}i,\ -\sqrt{3}+i,\ -1-\sqrt{3}i,\ \sqrt{3}-i$

2005 3 (c) (ii) $\cos^4\theta = \frac{1}{8}[\cos 4\theta + 4\cos 2\theta + 3]$

2004 3 (b) (i) $\frac{1}{2} - \frac{\sqrt{3}}{2}i$

2003 3 (c) (ii) 32

2002 3 (a) $2(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$