COMPLEX NUMBERS & MATRICES (Q 3, PAPER 1)

2011

3. **(a)** Express $\frac{1+2i}{2-i}$ in the form of $a+bi$, where $i^2=-1$.

(b) **(i)** Find the two complex numbers such that

$$(a+bi)^2=-3+4i.$$

(ii) Hence solve the equation

$$x^2+x+1-i=0.$$

(c) **(i)** Let A and B be 2×2 matrices, where A has an inverse.

Show that $(A^{-1}BA)^n=A^{-1}B^nA$ for all $n\in\mathbb{N}$.

Let $P=\begin{pmatrix}3 & 1\\ 5 & 2\end{pmatrix}$ and $M=\begin{pmatrix}-5 & 3\\ -10 & 6\end{pmatrix}$.

(ii) Evaluate $P^{-1}MP$ and hence $(P^{-1}MP)^n$.

(iii) Hence, or otherwise, show that $M^n=M$, for all $n\in\mathbb{N}$.

ANSWERS

3 (a) $0+i$

(b) (i) $\pm(1+2i)$ (ii) $x=-1-i,\ i$

(c) (ii) $P^{-1}MP=\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$, $(P^{-1}MP)^n=\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$