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COMPLEX NUMBERS & MATRICES (Q 3, PAPER 1)

2010

3   (a) Find x and y such that
$$\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 20 \\ 32 \end{pmatrix}.$$

(b) Let $z_1 = s + 8i$ and $z_2 = t + 8i$, where $s \in \mathbf{R}$, $t \in \mathbf{R}$, and $i^2 = -1$.

(i) Given that $|z_1| = 10$, find the possible values of s.

(ii) Given that $\arg(z_2) = \dfrac{3\pi}{4}$, find the value of t.

(c) (i) Use De Moivre's theorem to find, in polar form, the five roots of the equation
$$z^5 = 1.$$

(ii) Choose one of the roots w, where $w \neq 1$. Prove $w^2 + w^3$ that is real.

ANSWERS
3   (a) $x = 4$, $y = 2$
(b) (i) $s = \pm 6$ (ii) $t = -8$
(c) (i) $1, \cos\left(\dfrac{2\pi}{5}\right) + i\sin\left(\dfrac{2\pi}{5}\right)$, $\cos\left(\dfrac{4\pi}{5}\right) + i\sin\left(\dfrac{4\pi}{5}\right)$,

$\cos\left(\dfrac{6\pi}{5}\right) + i\sin\left(\dfrac{6\pi}{5}\right)$, $\cos\left(\dfrac{8\pi}{5}\right) + i\sin\left(\dfrac{8\pi}{5}\right)$.