COMPLEX NUMBERS & MATRICES (Q 3, PAPER 1)

2009

3 (a) $z_1 = a + bi$ and $z_2 = c + di$, where $i^2 = -1$.

Show that $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$, where $\overline{z}$ is the complex conjugate of z.

(b) Let $A = \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$.

(i) Express A^3 in the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, where $a, b \in \mathbf{Z}$.

(ii) Hence, or otherwise, find A^{17}.

(c) (i) Use De Moivre's theorem to prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

(ii) Hence, find $\int \sin^3\theta \, d\theta$.

ANSWERS

3 (b) (i) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ (ii) $\frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$

(c) (ii) $-\frac{3}{4}\cos\theta + \frac{1}{12}\cos 3\theta + c$