COMPLEX NUMBERS & MATRICES (Q 3, PAPER 1)

2006

3 (a) Given that $z = 2 + i$, where $i^2 = -1$, find the real number d such that $z + \frac{d}{z}$ is real.

3 (b) (i) Use matrix methods to solve the simultaneous equations

$$4x - 2y = 5$$
$$8x + 3y = -4$$

(ii) Find the two values of k which satisfy the matrix equation

$$(1 \quad k)\begin{pmatrix} 3 & 4 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ k \end{pmatrix} = 11$$

3 (c) (i) Express $-8 - 8\sqrt{3}i$ in the form $r(\cos\theta + i\sin\theta)$.

(ii) Hence find $(-8 - 8\sqrt{3}i)^3$.

(iii) Find the four complex number z such that $z^4 = -8 - 8\sqrt{3}i$. Give your answers in the form $a + bi$, with a and b fully evaluated.

ANSWERS

3 (a) $d = 5$

3 (b) (i) $x = \frac{1}{4}$, $y = -2$ (ii) $k = -4, 2$

3 (c) (i) $16(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3})$ (ii) 4096

(iii) $1 + \sqrt{3}i, -\sqrt{3} + i, -1 - \sqrt{3}i, \sqrt{3} - i$