COMPLEX NUMBERS & MATRICES (Q 3, PAPER 1)

2005

3 (a) Given that $A=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$, show that $A^3 = A^{-1}$.

3 (b) Solve the quadratic equation $2iz^2+(6+2i)z+(3-6i)=0$, where $i^2=-1$.

3 (c) (i) $z=\cos\theta+i\sin\theta$. Use De Moivre's theorem to show that $z^n+\dfrac{1}{z^n}=2\cos n\theta$, for $n\in \mathbf{N}$.

(ii) Expand $\left(z+\dfrac{1}{z}\right)^4$ and hence express $\cos^4\theta$ in terms of $\cos 4\theta$ and $\cos 2\theta$.

ANSWERS

3 (b) $z=\dfrac{1+3i}{2}, \dfrac{-3+3i}{2}$

3 (c) (ii) $\cos^4\theta=\frac{1}{8}[\cos 4\theta+4\cos 2\theta+3]$