COMPLEX NUMBERS & MATRICES (Q 3, PAPER 1)

2004

3 (a) Find the real numbers p and q such that $2(p+iq)+i(p-iq)=5+i$, where $i^2 = -1$.

3 (b) (i) $z_1 = \cos\dfrac{4\pi}{3}+i\sin\dfrac{4\pi}{3}$ and $z_2 = \cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}$. Evaluate $z_1 z_2$, giving your answer in the form $x+iy$.

(ii) $w_1 = a+ib$ and $w_2 = c+id$. Prove that $\overline{(w_1 w_2)} = (\overline{w_1})(\overline{w_2})$, where $\overline{w}$ is the complex conjugate w.

3 (c) Let $A = \begin{pmatrix} 1 & -3 \\ -1 & 2 \end{pmatrix}$ and $P = \begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$.

(i) Evaluate $A^{-1}PA$ and hence $(A^{-1}PA)^{10}$.

(ii) Use the fact that $(A^{-1}PA)^{10} = A^{-1}P^{10}A$ to evaluate P^{10}.

ANSWERS

3 (a) $p = 3,\ q = -1$

3 (b) (i) $\tfrac{1}{2} - \tfrac{\sqrt{3}}{2}i$

3 (c) (i) $\begin{pmatrix} 1 & 0 \\ 0 & 1024 \end{pmatrix}$ (ii) $\begin{pmatrix} 3070 & 3069 \\ -2046 & -2045 \end{pmatrix}$