

COMPLEX NUMBERS & MATRICES (Q 3, PAPER 1)

2002

3 (a) Express $-1 + \sqrt{3}i$ in the form $r(\cos\theta + i\sin\theta)$, where $i^2 = -1$.

3 (b) (i) Given that $z = 2 - i\sqrt{3}$, find the real number t such that $z^2 + tz$ is real.

(ii) w is a complex number such that $w\bar{w} - 2iw = 7 - 4i$, where \bar{w} is the complex conjugate of w .

Find two possible values of w . Express each in the form $p + qi$, where $p, q \in \mathbf{R}$.

3 (c) The following three statements are true whenever x and y are real numbers:

- $x + y = y + x$
- $xy = yx$
- If $xy = 0$ then either $x = 0$ or $y = 0$.

Investigate whether the statements are also true when x is the matrix $\begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix}$ and

y is the matrix $\begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix}$.

ANSWERS

3 (a) $2(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3})$

3 (b) (i) $t = -4$ (ii) $2 - 3i, 2 + i$