COMPLEX NUMBERS & MATRICES (Q 3, PAPER 1)

2001

3 (a) Let $u = \dfrac{1+3i}{3+i}$ where $i^2 = -1$.

(i) Express u is the form $a + ib$ where $a, b \in \mathbf{R}$.

(ii) Evaluate $|u|$.

3 (b) (i) Write the simultaneous equations

$$x - \sqrt{3}y = -2$$
$$\sqrt{3}x + y = 2\sqrt{3}$$

in the form $A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 2\sqrt{3} \end{pmatrix}$ where A is a 2×2 matrix.

(ii) Then, find A^{-1} and use it to solve the equations for x and y.

3 (c) (i) Write $\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} -2 & 3 \\ -4 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ in the form $ax^2 + bxy + cy^2$ where $a, b, c \in \mathbf{Z}$.

(ii) Show that $z^2 - 16$ is a factor of $z^3 + (1+i)z^2 - 16z - 16(1+i)$ and hence, find the three roots of $z^3 + (1+i)z^2 - 16z - 16(1+i) = 0$.

ANSWERS

3 (a) (i) $u = \frac{3}{5} + \frac{4}{5}i$ (ii) $|u| = 1$

3 (b) (i) $A = \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$ (ii) $A^{-1} = \frac{1}{4}\begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$; $x = 1,\ y = \sqrt{3}$

3 (c) (i) $-2x^2 - xy + 5y^2$ (ii) $z = 4, -4, -1 - i$