COMPLEX NUMBERS & MATRICES (Q 3, PAPER 1)

2000

3 (a) Given that $A = \begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 \\ -5 & -2 \end{pmatrix}$, find $B^{-1}A$.

3 (b) (i) Simplify $\left(\dfrac{-2+3i}{3+2i} \right)$ and hence, find the value of $\left(\dfrac{-2+3i}{3+2i} \right)^{9}$ where $i^2 = -1$.

(ii) Find the two complex numbers $a + ib$ such that
$$(a + ib)^2 = 15 - 8i.$$

3 (c) Use De Moivre's theorem

(i) to prove that $\cos 3\theta = 4\cos^3 \theta - 3\cos\theta$

(ii) to express $(-\sqrt{3} - i)^{10}$ in the form $2^n(1 - i\sqrt{k})$ where $n, k \in \mathbf{N}$.

ANSWERS

3 (a) $\begin{pmatrix} 4 & -1 \\ -11 & 1 \end{pmatrix}$

(b) (i) i, i

(ii) $\pm(4 - i)$

(c) (ii) $2^9(1 - \sqrt{3}i)$