

COMPLEX NUMBERS & MATRICES (Q 3, PAPER 1)

1999

3 (a) If $A = \begin{pmatrix} 2 & 1 \\ 5 & 4 \end{pmatrix}$, find A^{-1} .

3 (b) (i) Find a quadratic equation whose roots are $3 + i$ and $3 - i$, where $i^2 = -1$.

(ii) Let $P(z) = z^3 - kz^2 + 22z - 20$, $k \in \mathbf{R}$.

$3 + i$ is a root of the equation $P(z) = 0$.

Find the value of k .

Find the other two roots of the equation $P(z) = 0$.

3 (c) (i) Solve for w

$$\sqrt{5}|w| + iw = 3 + i.$$

Write your answers in the form $u + iv$, $u, v \in \mathbf{R}$.

(ii) Use De Moivre's theorem to find three roots of the equation $z^6 - 1 = 0$.

ANSWERS

3 (a) $\begin{pmatrix} \frac{4}{3} & -\frac{1}{3} \\ -\frac{5}{3} & \frac{2}{3} \end{pmatrix}$

(b) (i) $z^2 - 6z + 10 = 0$

(ii) $k = 5$; $3 - i$, 2

(c) (i) $1 + 2i$, $1 - \frac{1}{2}i$

(ii) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$, $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$, -1 , $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$, $\frac{1}{2} - \frac{\sqrt{3}}{2}i$, 1