

**COMPLEX NUMBERS & MATRICES (Q 3, PAPER 1)**

**1998**

3 (a) Express  $\sqrt{3} + i$  in the form  $r(\cos\theta + i\sin\theta)$ , where  $i^2 = -1$ .

(b) If  $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$  and  $B = \begin{pmatrix} 4 & 1 \\ 1 & -1 \end{pmatrix}$ , find  $AB$ .

Show that  $B^{-1}AB$  is of the form  $\begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}$ , where  $p, q \in \mathbf{N}_0$ .

(c) Let  $z = \cos\theta + i\sin\theta$ .

Express  $\frac{2}{1+z}$  in the form  $1 - i\tan(k\theta)$ ,  $k \in \mathbf{Q}$  and  $z \neq -1$ .

**ANSWERS**

3 (a)  $2(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$

(b)  $\begin{pmatrix} 24 & 1 \\ 6 & -1 \end{pmatrix}, \begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix}$

(c)  $k = \frac{1}{2}$