Complex Numbers & Matrices (Q 3, Paper 1)

1997

3 (a) If $A = \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$, find the matrix C such that $C = A(A - B)$.

(b) Let $P(z) = z^3 - (10+i)z^2 + (29+10i)z - 29i$, where $i^2 = -1$.

(i) Determine the real numbers a and b if

$P(z) = (z-i)(z^2 + az + b)$.

(ii) Plot on an argand diagram the solution set of $P(z) = 0$.

(c) (i) Let $\omega_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $\omega_2 = (\omega_1)^2$.

Verify that

$x^2 + xy + y^2 = (x - \omega_1 y)(x - \omega_2 y)$, where $x, y \in \mathbf{R}$.

(ii) Express $2(1 - i\sqrt{3})$ in the form $r(\cos\theta + i\sin\theta)$.

Using De Moivre's theorem find the values for

$[2(1-i\sqrt{3})]^{\frac{3}{2}}$

and write your answers in the form $p + qi$, $p, q \in \mathbf{R}$.

Answers

3 (a) $\begin{pmatrix} 10 & 12 \\ 18 & 20 \end{pmatrix}$

(b) (i) $a = -10$, $b = 29$ (ii) $i, 5 \pm 2i$

(c) $4(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}), 0 \pm 8i$