# Complex Numbers & Matrices (Q 3, Paper 1)

## 1996

3 (a) If $A = \begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 \\ -2 & 1 \end{pmatrix}$, find a matrix $M$ such that $M = BA^{-1}$.

(b) $P(z) = (z-2)(z^2 - 10z + 28)$.

(i) Plot on an Argand diagram the solution set of $P(z) = 0$.

(ii) Verify that the three points form an equilateral triangle.

(c) (i) $z_1 = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$ and $z_2 = 3(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$ where $i^2 = -1$.

Calculate $z_1 z_2$ in the form $x + iy$ where $x, y \in \mathbf{R}$.

(ii) $(2+3i)(a+ib) = -1+5i$. Express $a+ib$ in the form $r(\cos\theta + i\sin\theta)$ and hence, or otherwise, calculate $(a+ib)^{11}$.

**Answers**

3 (a) $M = \begin{pmatrix} 1 & 0 \\ -10 & 7 \end{pmatrix}$

(b) (i) $z = 2,\ 5 \pm \sqrt{3}i$

(c) (i) 0 + 6i

(ii) $1+i,\ -32+32i$