CIRCLE (Q 1, PAPER 2)

LESSON NO. 7: PARAMETRIC EQUATIONS

2003

1 (a) For all values of $t \in \mathbf{R}$, the point $\left(\frac{3-3t^2}{1+t^2}, \frac{6t}{1+t^2}\right)$ lies on the circle $x^2+y^2=r^2$.

Find r, the radius of the circle.

SOLUTION

1 (a)

As the point lies on the circle, you can substitute it into the equation of the circle.

$$x^2+y^2=r^2 \Rightarrow \left(\frac{3-3t^2}{1+t^2}\right)^2+\left(\frac{6t}{1+t^2}\right)^2=r^2$$

$$\Rightarrow \frac{9-18t^2+9t^4+36t^2}{(1+t^2)^2}=r^2 \Rightarrow \frac{9t^4+18t^2+9}{(1+t^2)^2}=r^2$$

$$\Rightarrow \frac{9(t^4+2t^2+1)}{(1+t^2)^2}=r^2 \Rightarrow \frac{9(t^2+1)^2}{(1+t^2)^2}=r^2 \Rightarrow 9=r^2$$

$$\therefore r=3$$

2002

1 (a) The following parametric equations define a circle: $x=4+3\cos\theta$, $y=-2+3\sin\theta$, where $\theta \in \mathbf{R}$. What is the Cartesian equation of the circle?

SOLUTION

1 (a)

STEPS
1. Isolate the trig functions.
2. Square both sides.
3. Add.
4. Put $\cos^2 t+\sin^2 t=1$.

Parametric Equations: $x=4+3\cos\theta$, $y=-2+3\sin\theta$

$$x-4=3\cos\theta \Rightarrow (x-4)^2=9\cos^2\theta$$
$$y+2=3\sin\theta \Rightarrow (y+2)^2=9\sin^2\theta$$

$$(x-4)^2+(y+2)^2=9(\cos^2\theta+\sin^2\theta)$$
$$\Rightarrow (x-4)^2+(y+2)^2=9$$