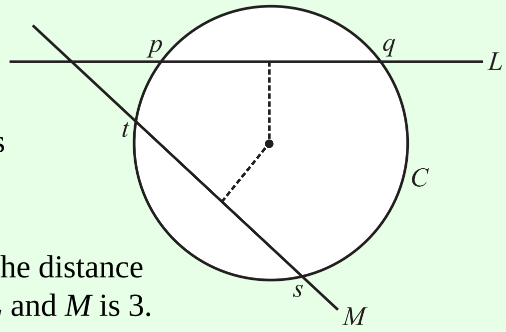
CIRCLE (Q 1, PAPER 2)

LESSON NO. 5: CHORDS

2002

1 (c) The circle C has equation $x^2 + y^2 - 4x + 6y - 12 = 0$. L intersects C at the points p and q. M intersects C at the points t and s. $|pq| = |ts| = 8$.



(i) Find the radius of C and hence show that the distance from the centre of C to each of the lines L and M is 3.

(ii) Given that L and M intersect at the point $(-4, 0)$, find the equations of L and M.

SOLUTION

1 (c) (i)

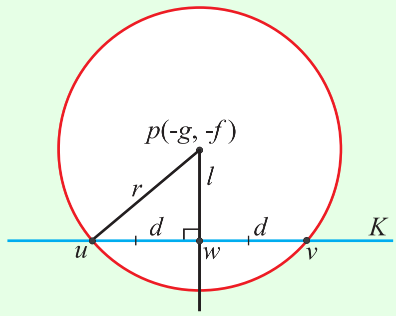
Circle C centre $(-g, -f)$, radius r.

$x^2 + y^2 + 2gx + 2fy + c = 0$ **3**

$r = \sqrt{g^2 + f^2 - c}$ **4**

SOME PROPERTIES OF CHORDS

1. The line K intersects the circle at points u and v.
2. [uv] is a chord.
3. The mid-point of the chord [uv] is w.
4. The line from the centre of the circle to w is perpendicular to the chord.
5. You can apply Pythagoras by completing a right-angled triangle.
6. The perpendicular distance of p to K is the distance l. Obviously, $l < r$.

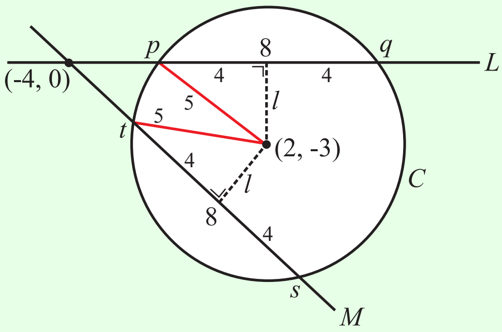


Circle C: $x^2 + y^2 - 4x + 6y - 12 = 0$

Centre $(2, -3)$, $r = \sqrt{4 + 9 + 12} = 5$

Apply Pythagoras to the right-angled triangles to show the distance l is 3.

$\therefore 4^2 + l^2 = 5^2 \Rightarrow l^2 = 25 - 16 = 9 \Rightarrow l = 3$



1 (c) (ii)

Equations of L and M: Point $(-4, 0)$, Slope $= +\frac{m}{1}$

$\Rightarrow mx - y + k = 0$

$\Rightarrow m(-4) - (0) + k = 0 \Rightarrow k = 4m$

$\Rightarrow mx - y + 4m = 0$ .....(**1**)

You know that the perpendicular distance from the centre to L and M is 3.

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$ **8**

$$3 = \frac{|m(2) - (-3) + 4m|}{\sqrt{m^2 + 1}} \Rightarrow 3\sqrt{m^2 + 1} = |6m + 3| \Rightarrow \sqrt{m^2 + 1} = |2m + 1|$$

$\Rightarrow m^2 + 1 = 4m^2 + 4m + 1 \Rightarrow 3m^2 + 4m = 0$

$\Rightarrow m(3m + 4) = 0 \Rightarrow m = 0, -\frac{4}{3}$

Substitute these values of m into equation **1** to give the two equations L and M.

$m = 0 \Rightarrow y = 0$

$m = -\frac{4}{3} \Rightarrow -\frac{4}{3}x - y + 4(-\frac{4}{3}) = 0 \Rightarrow 4x + 3y + 16 = 0$