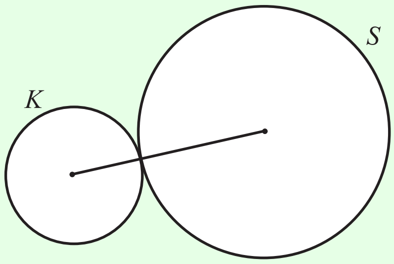
CIRCLE (Q 1, PAPER 2)

LESSON NO. 4: INTERSECTING CIRCLES

2005

1 (a) Circles S and K touch externally. Circle S has entre (8, 5) and radius 6. Circle K has centre $(2, -3)$. Calculate the radius of K.



SOLUTION

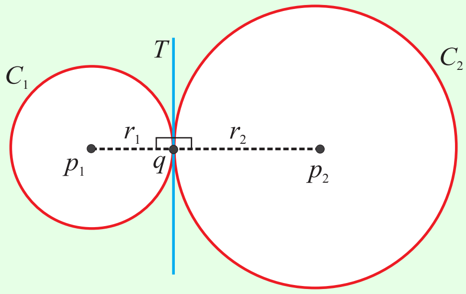
1 (a)

$$|p_1p_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(8-2)^2 + (5+3)^2}$$

$$= \sqrt{36 + 64} = 10$$

$$|p_1p_2| = r_1 + r_2 \Rightarrow 10 = 6 + r_2 \Rightarrow r_2 = 4$$

EXTERNAL TOUCH $|p_1p_2| = r_1 + r_2$



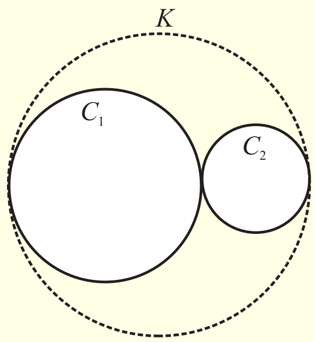
2003

1 (b) C_1: $x^2+y^2+2x-2y-23=0$ and

C_2: $x^2+y^2-14x-2y+41=0$ are two circles.

(i) Prove that C_1 and C_2 touch externally.

(ii) K is a third circle. Both C_1 and C_2 touch K internally. Find the equation of K.



SOLUTION

1 (b) (i)

Circle C centre $(-g,-f)$, radius r.

$x^2+y^2+2gx+2fy+c=0$ 3

$r=\sqrt{g^2+f^2-c}$ 4

C_1: $x^2+y^2+2x-2y-23=0$

Centre $p_1(-1, 1)$, $r_1=\sqrt{(-1)^2+(1)^2+23}=\sqrt{25}=5$

C_2: $x^2+y^2-14x-2y+41=0$

Centre $p_2(7, 1)$, $r_2=\sqrt{(7)^2+(1)^2-41}=\sqrt{9}=3$

EXTERNAL TOUCH $|p_1p_2|=r_1+r_2$

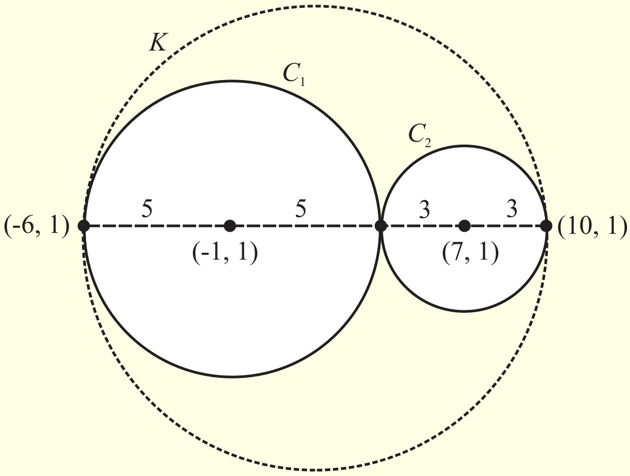
$|p_1p_2|=\sqrt{(-1-7)^2+(1-1)^2}=8$

$r_1+r_2=5+3=8$

Therefore, the two circles touch externally.

1 (b) (ii)

Copy the diagram of the circles. Notice that the diameters are parallel to the X-axis. They coincide with the line $y=1$. You can therefore easily find the co-ordinates of the endpoints of the diameter of K.



Centre of K: $\left(\frac{-6+10}{2}, \frac{1+1}{2}\right)=(2, 1)$

Radius of K: $r=8$

Circle C with centre (h, k), radius r.

$(x-h)^2+(y-k)^2=r^2$ 2

Equation of K: $(x-2)^2+(y-1)^2=8$