CIRCLE (Q 1, PAPER 2)

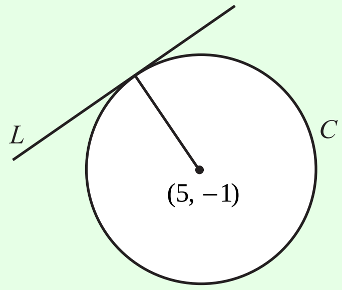
LESSON NO. 3: TANGENT AND CIRCLE

2006

1 (b) Circle C has centre $(5, -1)$. The line L: $3x-4y+11=0$ is a tangent to C.

(i) Show that the radius of C is 6.

(ii) The line $x + py + 1 = 0$ is also a tangent to C. Find two possible values of p.



SOLUTION

1 (b) (i)

The radius of the circle is the perpendicular distance from the centre to the tangent.

Centre $(5, -1)$, L: $3x-4y+11=0$

$$d=\frac{|3(5)-4(-1)+11|}{\sqrt{3^2+(-4)^2}}=\frac{|15+4+11|}{\sqrt{25}}=\frac{30}{5}=6$$

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$ 8

1 (b) (ii)

The perpendicular distance of the centre to this line is the radius (6 units).

Centre $(5, -1)$, L: $x+py+1=0$, $d=r=6$

$$\therefore 6=\frac{|5+p(-1)+1|}{\sqrt{1^2+p^2}}\Rightarrow 6\sqrt{p^2+1}=|6-p|$$

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$ 8

$\Rightarrow 36(p^2+1)=36-12p+p^2$ [Square both sides]

$\Rightarrow 36p^2+36=36-12p+p^2 \Rightarrow 35p^2+12p=0$

$\Rightarrow p(35p+12)=0 \Rightarrow p=0, -\frac{12}{35}$

2005

1 (b) (i) Prove that the equation of the tangent to the circle $x^2 + y^2 = r^2$ at the point (x_1, y_1) is $xx_1 + yy_1 = r^2$.

(ii) Hence, or otherwise, find the two values of b such that the line $5x + by = 169$ is a tangent to the circle $x^2 + y^2 = 169$.

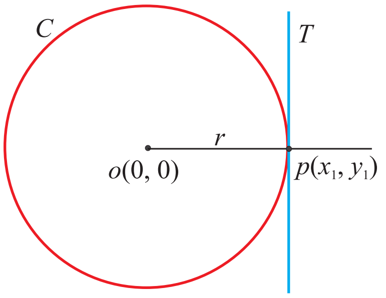
SOLUTION

1 (b) (i)

THE TANGENT THEOREM

STATEMENT: Prove that $xx_1 + yy_1 = r^2$ is the equation of the tangent to the circle $x^2 + y^2 = r^2$ at (x_1, y_1).

PROOF



Slope of $op = \frac{y_1}{x_1}$

$\therefore$ Slope of $T = -\frac{x_1}{y_1}$

$\therefore$ Equation of T: $xx_1 + yy_1 + k = 0$

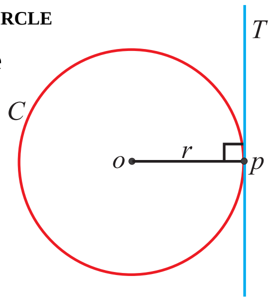
$(x_1, y_1) \in T \Rightarrow x_1^2 + y_1^2 + k = 0 \Rightarrow k = -x_1^2 - y_1^2 = -r^2$ since $(x_1, y_1) \in S$

$\therefore$ T: $xx_1 + yy_1 = r^2$

1 (b) (ii)

SOME POINTS YOU NEED TO KNOW ABOUT A TANGENT TO A CIRCLE

1. A tangent T intersects a circle C at one point only, the point of contact p.
2. The perpendicular distance from the centre of the circle C to the tangent T equals the radius r.
3. The tangent T is perpendicular to the line joining the centre o to the point of contact p.



Equation of tangent T: $xx_1 + yy_1 = r^2$ **5**

Method 1: Use point No. 2 above.

Circle: $x^2 + y^2 = 169$

Centre (0, 0), $r = 13$

T: $5x + by - 169 = 0$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$ **8**

$$\therefore 13 = \frac{|5(0) + b(0) - 169|}{\sqrt{b^2 + 25}} \Rightarrow 13\sqrt{b^2 + 25} = 169$$

$$\Rightarrow \sqrt{b^2 + 25} = 13 \Rightarrow b^2 + 25 = 169$$

$$\Rightarrow b^2 = 144 \Rightarrow b = \pm 12$$

Method 2: Use the equation of tangent formula.

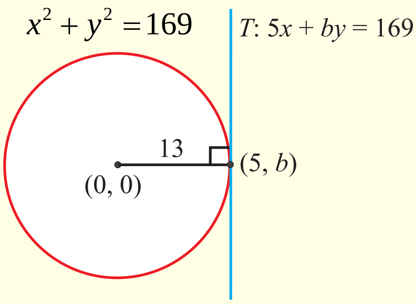
Equation of tangent T: $xx_1 + yy_1 = r^2$ **5**

T: $5x + by = 169$

Using the formula, you can see the point of contact is $(5, b)$.

Substitute this point into the circle and solve for b.

$x^2 + y^2 = 169 \Rightarrow 25 + b^2 = 169 \Rightarrow b^2 = 144 \Rightarrow b = \pm 12$



2003

1 (c) The line $ax + by = 0$ is a tangent to the circle $x^2 + y^2 - 12x + 6y + 9 = 0$ where $a, b \in \mathbf{R}$ and $b \neq 0$.

(i) Show that $\frac{a}{b} = -\frac{3}{4}$.

(ii) Hence, or otherwise, find the co-ordinates of the point of contact.

SOLUTION

1 (c) (i)

The perpendicular distance from the centre of the circle to the tangent equals the radius of the circle.

Circle: $x^2 + y^2 - 12x + 6y + 9 = 0$

Centre $(6, -3)$, $r = \sqrt{(6)^2 + (-3)^2 - 9} = \sqrt{36 + 9 - 9} = 6$

T: $ax + by = 0$

$\therefore 6 = \frac{|a(6) + b(-3)|}{\sqrt{a^2 + b^2}} \Rightarrow 6\sqrt{a^2 + b^2} = |6a - 3b|$

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ **8**

$\Rightarrow 2\sqrt{a^2 + b^2} = |2a - b|$ [Square both sides.]

$\Rightarrow 4a^2 + 4b^2 = 4a^2 - 4ab + b^2 \Rightarrow 3b^2 = -4ab$

$\Rightarrow 3b = -4a \Rightarrow \frac{a}{b} = -\frac{3}{4}$

1 (c) (ii)

$ax + by = 0 \Rightarrow x = -\frac{b}{a}y = \frac{4}{3}y$ [Using the previous result]

Substitute this value of x into the circle equation.

$x^2 + y^2 - 12x + 6y + 9 = 0 \Rightarrow (\frac{4}{3}y)^2 + y^2 - 12(\frac{4}{3}y) + 6y + 9 = 0$

$\Rightarrow \frac{16}{9}y^2 + y^2 - 16y + 6y + 9 = 0 \Rightarrow 16y^2 + 9y^2 - 90y + 81 = 0$

$\Rightarrow 25y^2 - 90y + 81 = 0 \Rightarrow (5y - 9)(5y - 9) = 0 \Rightarrow y = \frac{9}{5}$

$x = \frac{4}{3}y = \frac{4}{3}(\frac{9}{5}) = \frac{12}{5}$

Ans: $(\frac{12}{5}, \frac{9}{5})$