CIRCLE (Q 1, PAPER 2)

2010

1 (a) A circle with centre (3, –4) passes through the point (7, –3). Find the equation of the circle.

(b) (i) Find the centre and radius of the circle $x^2+y^2-8x-10y+32=0$.

(ii) The line $3x + 4y + k = 0$ is a tangent to the circle $x^2+y^2-8x-10y+32=0$. Find the two possible values of k.

(c) A circle has the line $y = 2x$ as a tangent at the point (2, 4). The circle also passes through the point (4, –2). Find the equation of the circle.

SOLUTION

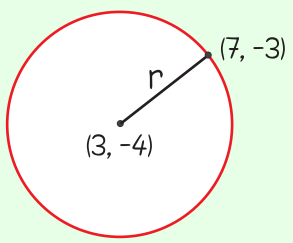
1 (a)

Centre (3, –4)

$r=\sqrt{(7-3)^2+(-3+4)^2}=\sqrt{16+1}=\sqrt{17}$

$(x-h)^2+(y-k)^2=r^2$

Equation of circle: $(x-3)^2+(y+4)^2=17$



1 (b) (i)

$x^2+y^2-8x-10y+32=0$

$g=-4,\ f=-5,\ c=32$

Centre: $(-g,-f)=(4, 5)$

$r=\sqrt{(-4)^2+(-5)^2-32}=\sqrt{16+25-32}=\sqrt{9}=3$

Circle C with centre $(-g,-f)$, radius r.

$x^2+y^2+2gx+2fy+c=0$

$r=\sqrt{g^2+f^2-c}$

1 (b) (ii)

The perpendicular distance of the centre to the tangent is equal to the radius.

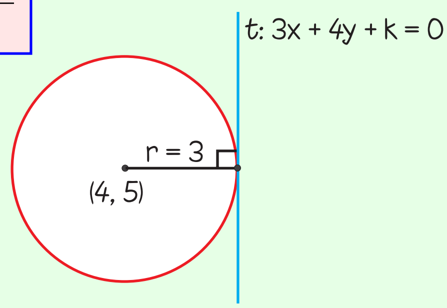
$3=\frac{|3(4)+4(5)+k|}{\sqrt{3^2+4^2}}\Rightarrow 3=\frac{|12+20+k|}{\sqrt{25}}$

$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$15=|k+32|$

$\pm 15=k+32$

$\therefore k=-47,\ -17$



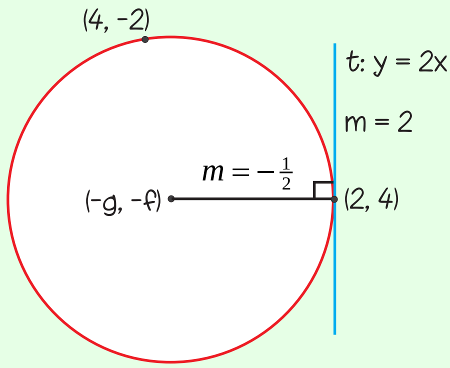
1 (c)

$(4, -2) \in c \Rightarrow 4^2 + (-2)^2 + 2g(4) + 2f(-2) + c = 0$

$16 + 4 + 8g - 4f + c = 0$

$8g - 4f + c = -20$.....**(1)**

$x^2 + y^2 + 2gx + 2fy + c = 0$



$(2, 4) \in c \Rightarrow 2^2 + (4)^2 + 2g(2) + 2f(4) + c = 0$

$4 + 16 + 4g + 8f + c = 0$

$4g + 8f + c = -20$....**(2)**

$8g - 4f + c = -20$.....**(1)**

$4g + 8f + c = -20$....**(2)**

$4g - 12f \quad = 0 \Rightarrow g = 3f$

Centre $(-g, -f) = (-3f, -f)$

Slope between centre and point of contact:

$$\frac{4+f}{2+g} = -\frac{1}{2} \Rightarrow 8 + 2f = -2 - g$$

$g + 2f = -10$

$(3f) + 2f = 10 \Rightarrow 5f = 10 \Rightarrow f = 2$

$\therefore g = 6$

Centre $(-g, -f) = (-6, -2)$

$$r = \sqrt{(2+6)^2 + (4+2)^2} = \sqrt{64+36} = \sqrt{100} = 10$$

Equation of circle: $(x-6)^2 + (y-2)^2 = 20$ $(x-h)^2 + (y-k)^2 = r^2$