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CIRCLE (Q 1, PAPER 2)

2008

1 (a) A circle with centre $(-3, 2)$ passes through the point $(1, 3)$.
Find the equation of the circle.

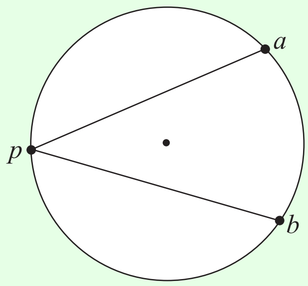
(b) (i) Prove that the equation of the tangent to the circle $x^2 + y^2 = r^2$ at the point (x_1, y_1) is $xx_1 + yy_1 = r^2$.

(ii) A tangent is drawn to the circle $x^2 + y^2 = 13$ at the point $(2, 3)$.
This tangent crosses the x-axis at $(k, 0)$. Find the value of k.

(c) A circle passes through the points $a(8, 5)$ and $b(9, -2)$.
The centre of the circle lies on the line $2x - 3y - 7 = 0$.

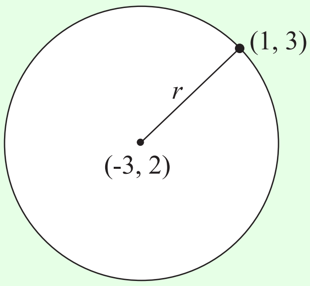
(i) Find the equation of the circle.

(ii) p is a point on the major arc ab of the circle.
Show that $|\angle apb| = 45^\circ$.



SOLUTION

1 (a)



Circle C with centre (h, k), radius r.

$(x-h)^2 + (y-k)^2 = r^2$ 2

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ 1

$r = \sqrt{(-3-1)^2 + (2-3)^2}$

$\Rightarrow r = \sqrt{16+1} = \sqrt{17}$

Equation of the circle: $(x-(-3))^2 + (y-2)^2 = (\sqrt{17})^2$

$\therefore (x+3)^2 + (y-2)^2 = 17$

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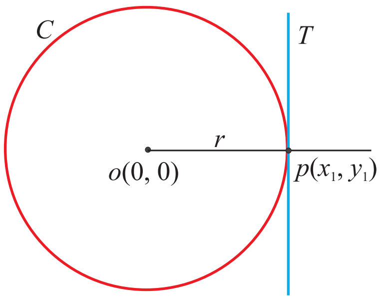
1 (b) (i)

THE TANGENT THEOREM

STATEMENT: Prove that $xx_1 + yy_1 = r^2$ is the equation of the tangent to the circle $x^2 + y^2 = r^2$ at (x_1, y_1).



PROOF

Slope of $op = \frac{y_1}{x_1}$

$\therefore$ Slope of $T = -\frac{x_1}{y_1}$

$\therefore$ Equation of T: $xx_1 + yy_1 + k = 0$

$(x_1, y_1) \in T \Rightarrow x_1^2 + y_1^2 + k = 0 \Rightarrow k = -x_1^2 - y_1^2 = -r^2$ since $(x_1, y_1) \in C$

$\therefore$ T: $xx_1 + yy_1 = r^2$

1 (b) (ii)

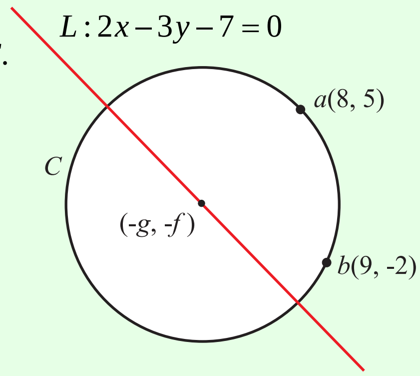
Using the tangent theorem, the equation of this tangent is $2x + 3y = 13$.
To find out where it cuts the x-axis, put $y = 0$.

$2x + 0 = 13 \Rightarrow x = \frac{13}{2}$

The x-intercept is $(\frac{13}{2}, 0) \Rightarrow k = \frac{13}{2}$

1 (c) (i)

$x^2 + y^2 + 2gx + 2fy + c = 0$ **3**



Substitute points a and b into the equation of the circle, C.

$a \in C \Rightarrow 64 + 25 + 16g + 10f + c = 0$

$\therefore 16g + 10f + c = -89$.....**(1)**

$b \in C \Rightarrow 81 + 4 + 18g - 4f + c = 0$

$\therefore 18g - 4f + c = -85$.....**(2)**

$(-g, -f) \in L \Rightarrow -2g + 3f - 7 = 0$

$\therefore 2g - 3f = -7$....**(3)**

Eliminate c by subtracting Eqn. (**1**) from Eqn. (**2**):

$\therefore 2g - 14f = 4$.....**(4)**

Now subtract Eqn. (**4**) from Eqn. (**3**):

$\therefore 11f = -11 \Rightarrow f = -1$

Substitute this value of f into Eqn. (**3**):

$\therefore 2g - 3(-1) = -7 \Rightarrow 2g = -10$

$\therefore g = -5$

Substitute these values of g and f into Eqn. (**1**):

$\therefore 16(-5) + 10(-1) + c = -89 \Rightarrow -80 - 10 + c = -89$

$\therefore c = 1$

Equation of C: $x^2 + y^2 - 10x - 2y + 1 = 0$

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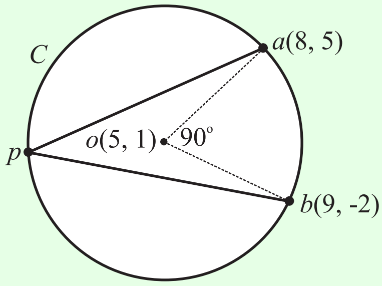
1 (c) (ii)

If $|\angle apb| = 45^\circ$ then the angle standing at the centre must be 90°. Therefore, you need to show that *ao* is perpendicular to *bo*, where *o* is the centre of the circle.



Slope of *ao*: $m_1 = \frac{5-1}{8-5} = \frac{4}{3}$

Slope of *bo*: $m_2 = \frac{-2-1}{9-5} = -\frac{3}{4}$

The slopes are perpendicular as $m_1 \times m_2 = -1$.

$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$ 2