Circle (Q 1, Paper 2)

2000

1 (a) The equation of a circle is $x^2 + y^2 = 130$.

Find the slope of the tangent to the circle at the point $(-7, 9)$.

1 (b) $x^2 + y^2 - 6x + 4y - 12 = 0$ is the equation of a circle.

Write down the coordinates of its centre and the length of its radius.

$x^2 + y^2 + 12x - 20y + k = 0$ is another circle, where $k \in \mathbf{R}$.

The two circles touch externally. Find the value of k.

1 (c) A circle intersects a line at the points $a(-3, 0)$ and $b(5, -4)$.

The midpoint of $[ab]$ is m. Find the coordinates of m.

The distance from the centre of the circle to m is $\sqrt{5}$.

Find the equations of the two circles that satisfy these conditions.

Solution

1 (a)

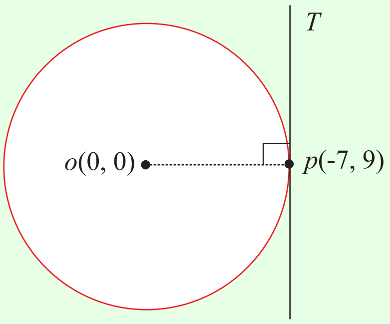
$m = \tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$ **2**

Circle C with centre $(0, 0)$, radius r.

$x^2 + y^2 = r^2$ **1**

Slope of op: $m = \dfrac{9-0}{-7-0} = -\dfrac{9}{7}$

Slope of Tangent: $m^{\perp} = \frac{7}{9}$

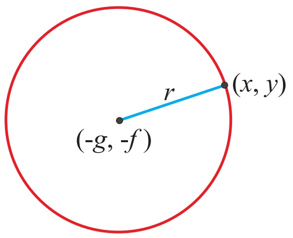


1 (b)

Circle C with centre $(-g, -f)$, radius r.

$x^2 + y^2 + 2gx + 2fy + c = 0$ **3**

$r = \sqrt{g^2 + f^2 - c}$ **4**



$x^2 + y^2 - 6x + 4y - 12 = 0$

Centre $(3, -2)$, $r = \sqrt{9+4+12} = \sqrt{25} = 5$

$x^2 + y^2 + 12x - 20y + k = 0$

Centre $(-6, 10)$, $r = \sqrt{36+100-k} = \sqrt{136-k}$

$|p_1p_2| = r_1 + r_2$

$p_1(3, -2),\ r_1 = 5$

$p_2(-6, 10),\ r_2 = \sqrt{136-k}$

$\therefore \sqrt{(-6-3)^2 + (10-(-2))^2} = 5 + \sqrt{136-k}$

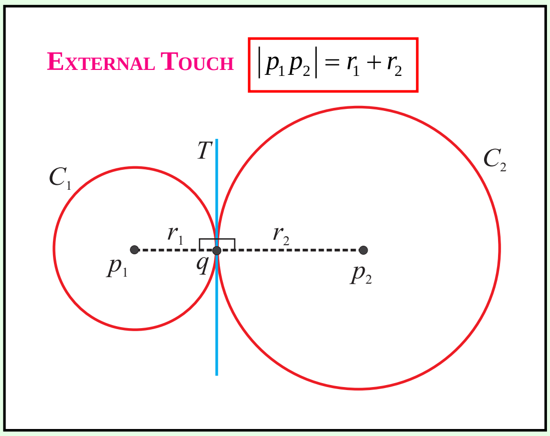
$\Rightarrow \sqrt{81+144} = 5 + \sqrt{136-k}$

$\Rightarrow 15 = 5 + \sqrt{136-k}$

$\Rightarrow 10 = \sqrt{136-k}$

$\Rightarrow 100 = 136 - k$

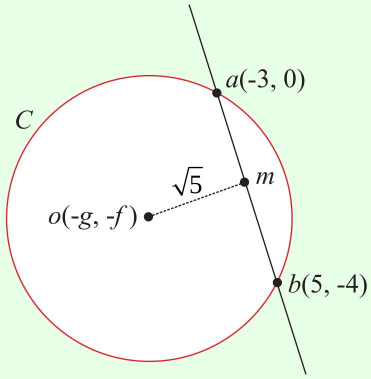
$\therefore k = 36$



1 (c)

$a(-3, 0),\ b(5, -4)$

$\Rightarrow m\left(\frac{-3+5}{2}, \frac{0-4}{2}\right) = m(1, -2)$



$x^2 + y^2 + 2gx + 2fy + c = 0$ **3**

$a(-3, 0) \in C \Rightarrow 9 + 0 - 6g + 0 + c = 0$

$\Rightarrow -6g + c = -9$.......(**1**)

$b(5, -4) \in C \Rightarrow 25 + 16 + 10g - 8f + c = 0$

$\Rightarrow 10g - 8f + c = -41$.......(**2**)

$|om| = \sqrt{5} \Rightarrow \sqrt{(1+g)^2 + (-2+f)^2} = \sqrt{5}$

$(1+g)^2 + (-2+f)^2 = 5$....(**3**)

Using the 3 equations, solve for *g*, *f* and *c*.

Combine Eqns (**1**) and (**2**).

$-6g \quad + c = -9$.....(**1**)$(\times -1)$

$10g - 8f + c = -41$...(**2**)

→

$6g \quad - c = 9$

$10g - 8f + c = -41$

$16g - 8f \quad = -32 \Rightarrow 2g - f = -4 \Rightarrow f = 2g + 4$

Substitute this value of f into Eqn. (**3**).

$(1+g)^2 + (-2+2g+4)^2 = 5$

$\Rightarrow (1+g)^2 + (2g+2)^2 = 5$

$\Rightarrow 1 + 2g + g^2 + 4g^2 + 8g + 4 = 5$

$\Rightarrow 5g^2 + 10g = 0$

$\Rightarrow g^2 + 2g = 0$

$\Rightarrow g(g+2) = 0$

$\therefore g = 0, -2$

Now $f = 2g + 4$

$\therefore\ f = 4,\ 0$

Substitute the values of g into Eqn (**1**) to find c.

$g = 0 \Rightarrow c = -9$

$g = -2 \Rightarrow 12 + c = -9 \Rightarrow c = -21$

Equations of circles:

$C_1 : g = 0,\ f = 4,\ c = -9$

$x^2 + y^2 + 8y - 9 = 0$

$C_2 : g = -2,\ f = 0,\ c = -21$

$x^2 + y^2 - 4x - 21 = 0$