Circle (Q 1, Paper 2)

1998

1 (a) $p(k, 2)$ and $q(-6, -k)$ are the end points of a diameter of a circle S with centre $(3, -5)$.
Find the value of k.
Verify that the radius length of S is $\sqrt{130}$.

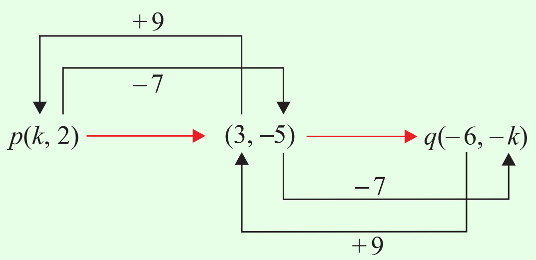
(b) K is the circle with equation $x^2 + y^2 = 100$.
Show, by calculation, that the point $a(12, -9)$ lies outside K.
Find the equation of the line oa, where o is the origin.
Find the coordinates of the points where oa intersects K.

(c) A circle of radius length $\sqrt{20}$ contains the point $(-1, 3)$. Its centre lies on the line $x + y = 0$.
Find the equations of the two circles that satisfy these conditions.

Solution

1 (a)

$p(k, 2) \to (3, -5) \to q(-6, -k)$



$\therefore p(12, 2) \to (3, -5) \to q(-6, -12)$

$\therefore k = 12$

$r = \sqrt{(3-(-6))^2 + (-5-(-12))^2} = \sqrt{9^2 + 7^2}$

$\Rightarrow r = \sqrt{81+49}$

$\therefore r = \sqrt{130}$

1 (b)

Is a point on a circle, inside a circle or outside a circle?
Substitute the point into the circle.
On the circle: Both sides are equal.
Inside the circle: The left hand side is less than the right hand side.
Outside the circle: The left hand side is greater than the right hand side.

$(12)^2 + (-9)^2 = 144 + 81 = 125 > 100$

Therefore, a lies outside K.

$o(0,\ 0),\ a(12,\ -9)$

$\therefore m = \frac{-9-0}{12-0} = \frac{-9}{12} = -\frac{3}{4}$ $\quad$ $m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$ **2**

Equation of oa: $3x + 4y + k = 0$ $\quad$ $y - y_1 = m(x - x_1)$ **3**

$(0,\ 0) \in oa \Rightarrow 3(0) + 4(0) + k = 0 \Rightarrow k = 0$

$\therefore 3x + 4y = 0$

Steps

1. Isolate x or y using equation of the line.
2. Substitute into the equation of the circle and solve simultaneously.

1. $3x + 4y = 0 \Rightarrow x = -\frac{4}{3}y$

2. $x^2 + y^2 = 100 \Rightarrow (-\frac{4}{3}y)^2 + y^2 = 100$

$\Rightarrow \frac{16}{9}y^2 + y^2 = 100$

$\Rightarrow 16y^2 + 9y^2 = 900$

$\Rightarrow 25y^2 = 900 \Rightarrow y^2 = 36$

$\Rightarrow y = \pm 6$

$y = 6 : x = -\frac{4}{3}(6) = -8$

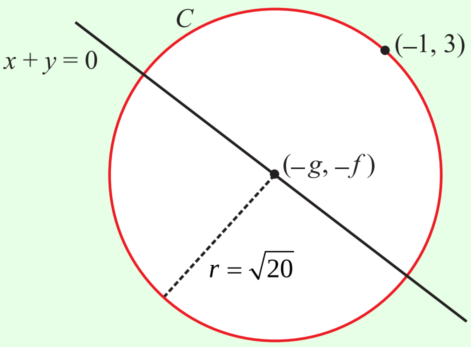
$y = -6 : x = -\frac{4}{3}(-6) = 8$

$\therefore (8,\ -6),\ (-8,\ 6)$ are the points of intersection.

1 (c)

$x^2 + y^2 + 2gx + 2fy + c = 0$ **3**

$r = \sqrt{g^2 + f^2 - c}$ **4**



$(-1,\ 3) \in C \Rightarrow (-1)^2 + (3)^2 + 2g(-1) + 2f(3) + c = 0$

$\Rightarrow 1 + 9 - 2g + 6f + c = 0$

$\therefore -2g + 6f + c = -10$....**(1)**

$(-g,\ -f) \in x + y = 0$

$\Rightarrow -g - f = 0$

$\therefore g = -f$......**(2)**

$r = \sqrt{20} \Rightarrow \sqrt{20} = \sqrt{g^2 + f^2 - c}$

$\Rightarrow g^2 + f^2 - c = 20$.....**(3)**

Now combine Equations **1**, **2** and **3** to solve for g, f and c.

Substitute Eqn. (**2**) into Eqns. (**1**) and (**3**).

$g = -f \Rightarrow -2(-f) + 6f + c = -10$

$\Rightarrow 2f + 6f + c = -10$

$\Rightarrow 8f + c = -10 \ldots(\mathbf{4})$

$g = -f \Rightarrow (-f)^2 + f^2 - c = 20$

$\Rightarrow f^2 + f^2 - c = 20$

$\therefore 2f^2 - c = 20 \ldots.(\mathbf{5})$

Add Eqns. (**4**) and (**5**):

$\therefore 2f^2 + 8f = 10$

$\Rightarrow f^2 + 4f - 5 = 0$

$\Rightarrow (f+5)(f-1) = 0$

$\therefore f = -5,\ f = 1$

$\therefore g = 5,\ g = -1$

$\therefore c = 30,\ c = -18$

Equations of 2 circles:

$x^2 + y^2 + 10x - 10y + 30 = 0$

$x^2 + y^2 - 2x + 2y - 18 = 0$