CIRCLE (Q 1, PAPER 2)

1997

1 (a) The equation of a circle is

$(x+7)(x+3)+(y-2)(y+2)=0.$

Find the centre and radius length of the circle.

(b) Prove that the equation of the tangent to the circle $x^2+y^2=r^2$ at the point (x_1, y_1) on the circle is

$xx_1+yy_1=r^2.$

(c) The x axis is a tangent to the circle $x^2+y^2+2gx+2fy+c=0.$

Show that

$g^2=c.$

The x axis is a tangent to a circle K at the point (3, 0).

The point $(-1, 4) \in K.$

Find the equation of $K.$

SOLUTION

1 (a)

$x^2+y^2+2gx+2fy+c=0$ 3

$r=\sqrt{g^2+f^2-c}$ 4

$(x+7)(x+3)+(y-2)(y+2)=0$

$\Rightarrow x^2+10x+21+y^2-4=0$

$\Rightarrow x^2+y^2+10x+17=0$

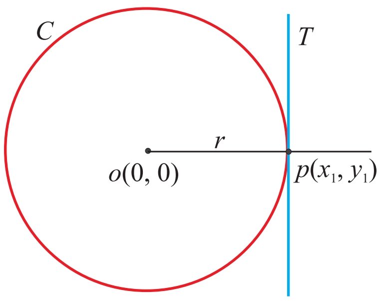
$\therefore$ Centre $(-g, -f)=(-5, 0)$

$\therefore r=\sqrt{(-5)^2+(0)^2-17}=\sqrt{25-17}=\sqrt{8}$

1 (b)

THE TANGENT THEOREM

STATEMENT: Prove that $xx_1+yy_1=r^2$ is the equation of the tangent to the circle $x^2+y^2=r^2$ at (x_1, y_1).



PROOF

Slope of $op = \frac{y_1}{x_1}$

$\therefore$ Slope of $T = -\frac{x_1}{y_1}$

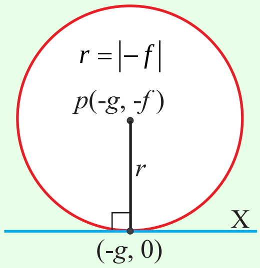
$\therefore$ Equation of T: $xx_1+yy_1+k=0$

$(x_1, y_1)\in T \Rightarrow x_1^2+y_1^2+k=0 \Rightarrow k=-x_1^2-y_1^2=-r^2$ since $(x_1, y_1)\in C$

$\therefore$ T: $xx_1+yy_1=r^2$

1 (c)

$r=\sqrt{g^2+f^2-c}$ **4**



$\therefore r^2 = f^2 \Rightarrow f^2 = g^2 + f^2 - c$

$\therefore g^2 = c$....(**1**)

$x^2+y^2+2gx+2fy+c=0$ **3**

$(3,\ 0)\in K \Rightarrow (3)^2+(0)^2+2g(3)+2f(0)+c=0$

$\Rightarrow 9+6g+c=0$

$\therefore 6g+c=-9$....(**2**)

$(-1,\ 4)\in K \Rightarrow (-1)^2+(4)^2+2g(-1)+2f(4)+c=0$

$\Rightarrow 1+16-2g+8f+c=0$

$\therefore -2g+8f+c=-17$....(**3**)

Now combine Equations **1**, **2** and **3** to solve for *g*, *f* and *c*.

Substitute Eqn. (**1**) into Eqn. (**2**).

$g^2=c \Rightarrow 6g+g^2=-9$

$\Rightarrow g^2+6g+9=0$

$\Rightarrow (g+3)(g+3)=0$

$\therefore g=-3$

$\therefore c=9$

Now substitute these values for *g* and *c* into Eqn. (**3**).

$-2g+8f+c=-17 \Rightarrow -2(-3)+8f+9=-17$

$\Rightarrow 6+8f+26=0$

$\Rightarrow 8f=-32$

$\therefore f=-4$

$K: x^2+y^2-6x-8y+9=0$