CIRCLE (Q 1, PAPER 2)

LESSON NO. 1: THE THREE CIRCLE EQUATIONS

2006

1 (a) $a(-1, -3)$ and $b(3, 1)$ are the end-points of a diameter of a circle. Write down the equation of a circle.

SOLUTION

1 (a)

The centre o is the mid-point of $[ab]$.

Mid-point $=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=\left(\frac{-1+3}{2}, \frac{-3+1}{2}\right)=(1, -1)$

The radius of the circle is half the distance $|ab|$.

$r=\frac{1}{2}\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\frac{1}{2}\sqrt{(3+1)^2+(1+3)^2}=\frac{1}{2}\sqrt{32}=2\sqrt{2}$

Circle C with centre (h, k), radius r. $(x-h)^2+(y-k)^2=r^2$ **2**

C: $(x-1)^2+(y+1)^2=(2\sqrt{2})^2 \Rightarrow (x-1)^2+(y+1)^2=8$. This answer is fine. However, if you decide to expand the equation you will get: $x^2+y^2-2x+2y-6=0$

2004

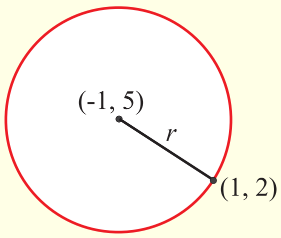
1 (a) A circle has centre $(-1, 5)$ and passes through the point $(1, 2)$. Find the equation of the circle.

SOLUTION

1 (a)

Circle C with centre (h, k), radius r.

$(x-h)^2+(y-k)^2=r^2$ **2**



Centre $(-1, 5)$, $r=\sqrt{(-1-1)^2+(5-2)^2}=\sqrt{4+9}=\sqrt{13}$

Circle: $(x+1)^2+(y-5)^2=13$

Multiplying this equation also gives $x^2+y^2+2x-10y+13=0=13$

2002

1 (b) The points $a(-2, 4)$, $b(0, -10)$ and $c(6, -2)$ are the vertices of a triangle.

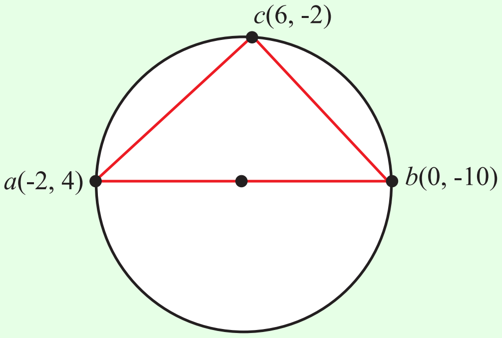
(i) Verify the the triangle is right-angled at c.

(ii) Hence, or otherwise, find the equation of the circle that passes through the points a, b and c.

SOLUTION

1 (b) (i)

To show $ac \perp bc$



$m = \tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$ 2

Slope of ac: $m_1 = \dfrac{4+2}{-2-6} = \dfrac{6}{-8} = -\dfrac{3}{4}$

Slope of bc: $m_2 = \dfrac{-10+2}{0-6} = \dfrac{-8}{-6} = \dfrac{4}{3}$

Two lines are perpendicular if the product of their slopes is -1.
$K \perp L \Leftrightarrow m_1 \times m_2 = -1$.

$m_1 \times m_2 = (-\tfrac{3}{4})(\tfrac{4}{3}) = -1 \Rightarrow ac \perp bc$

1 (b) (ii)

The best way to do this question is to find the centre and radius of the circle from the points given. This is easily done if you remember a theorem from your Junior Cert, i.e. the angle standing on the diameter of a circle is a right-angle. As $\angle acb$ is a right angle, it follows that [ab] is the diameter of the circle.
Therefore, the centre is the midpoint of [ab].

Centre: $\left(\dfrac{-2+0}{2}, \dfrac{4-10}{2}\right) = (-1, -3)$

The radius is the distance from the centre to any point, say a.

$r = \sqrt{(-2+1)^2 + (4+3)^2} = \sqrt{1+49} = \sqrt{50}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ 1

Equation of circle: Centre $(-1, -3)$, $r = \sqrt{50}$

$(x+1)^2 + (y+3)^2 = 50$

Circle C with centre (h, k), radius r.

$(x-h)^2 + (y-k)^2 = r^2$ 2

This answer is fine. If you multiply this equation out you get: $x^2 + y^2 + 2x + 6y - 40 = 0$

2001

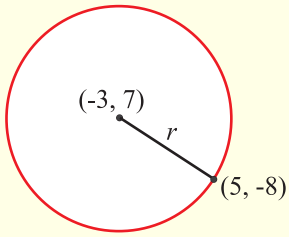
1 (a) A circle with centre $(-3, 7)$ passes through the point $(5, -8)$. Find the equation of the circle.

SOLUTION

1 (a)

Circle C with centre (h, k), radius r.

$(x-h)^2+(y-k)^2=r^2$ **2**



$r=\sqrt{(-3-5)^2+(7+8)^2}=\sqrt{64+225}=\sqrt{289}$

Equation of circle: $(x+3)^2+(y-7)^2=289$

This answer is fine, but you can multiply it out to get: $x^2+y^2+6x-14y-231=0$