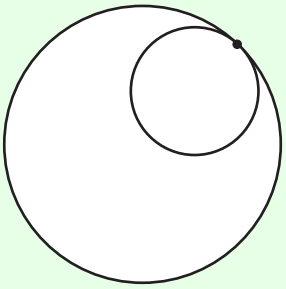
CIRCLE (Q 1, PAPER 2)

2007

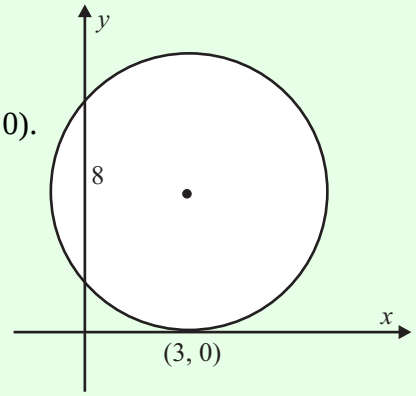
1 (a) The following parametric equations define a circle:
$x = 5 + 7\cos\theta,\ y = 7\sin\theta,$ where $\theta \in \mathbf{R}$.
What is the Cartesian equation of the circle?



(b) $x^2 + y^2 - 4x - 6y + 5 = 0$ and $x^2 + y^2 - 6x - 8y + 23 = 0$ are two circles.
(i) Prove that the circles touch internally.

(ii) Find the coordinates of the point of contact of the two circles.

(c) A circle has its centre in the first quadrant.
The x-axis is a tangent to the circle at the point (3, 0).
The circle cuts the y-axis at points that are 8 units apart.
Find the equation of the circle.



1 (a)

$x = 5 + 7\cos\theta \Rightarrow x - 5 = 7\cos\theta \Rightarrow (x-5)^2 = 49\cos^2\theta$

$y = 7\sin\theta \Rightarrow y^2 = 49\sin^2\theta$

$\therefore (x-5)^2 + y^2 = 49(\cos^2\theta + \sin^2\theta)$

$\therefore (x-5)^2 + y^2 = 49$

STEPS
1. Isolate the trig functions.
2. Square both sides.
3. Add.
4. Put $\cos^2 t + \sin^2 t = 1$.

1 (b)

C_1: $x^2 + y^2 - 4x - 6y + 5 = 0$

Centre $p_1(2, 3)$, $r_1 = \sqrt{4+9-5} = \sqrt{8} = 2\sqrt{2}$

C_2: $x^2 + y^2 - 6x - 8y + 23 = 0$

Centre $p_2(3, 4)$, $r_2 = \sqrt{9+16-23} = \sqrt{2}$

Circle C with centre $(-g, -f)$, radius r.

$x^2 + y^2 + 2gx + 2fy + c = 0$ 3

$r = \sqrt{g^2 + f^2 - c}$ 4

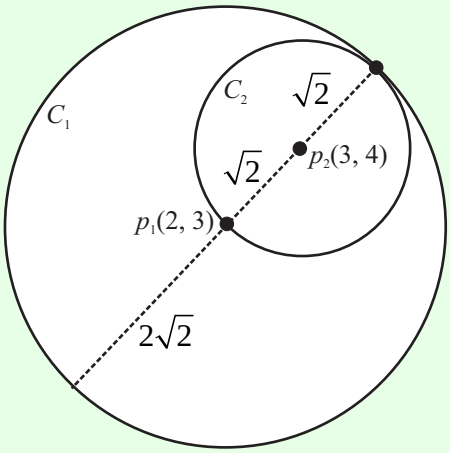
1 (b) (i)

INTERNAL TOUCH $|p_1p_2| = r_1 - r_2$

$|p_1p_2| = \sqrt{(2-3)^2 + (3-4)^2} = \sqrt{2}$

$r_1 - r_2 = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

Therefore, the circles touch internally.



1 (b) (ii)

As can be seen from the diagram, the centre of C_1 lies on C_2 because its radius is twice that of C_2. The point (3, 4) is the mid-point of (2, 3) and the point of contact.

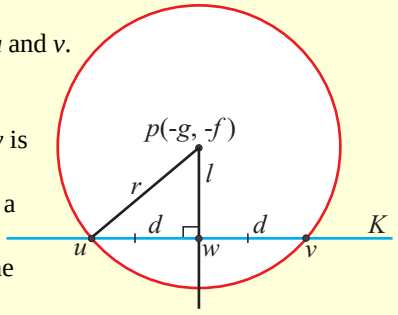
$(2,3) \rightarrow (3,4) \rightarrow (4,5)$

(4, 5) is the point of contact between the two circles.

1 (c)

SOME PROPERTIES OF CHORDS

1. The line K intersects the circle at points u and v.
2. $[uv]$ is a chord.
3. The mid-point of the chord $[uv]$ is w.
4. The line from the centre of the circle to w is perpendicular to the chord.
5. You can apply Pythagoras by completing a right-angled triangle.
6. The perpendicular distance of p to K is the distance l. Obviously, $l < r$.



Centre (3, 5), $r = 5$

Circle C with centre (h, k), radius r.

$(x-h)^2 + (y-k)^2 = r^2$ 2

Eqn. of circle: $(x-3)^2 + (y-5)^2 = 25$