CIRCLE (Q 1, PAPER 2)

2004

1 (a) A circle has centre $(-1, 5)$ and passes through the point (1, 2). Find the equation of the circle.

1 (b) The point $a(5, 2)$ is on the circle K: $x^2 + y^2 + px - 2y + 5 = 0$.

(i) Find the value of p.

(ii) The line L: $x - y - 1 = 0$ intersects the circle K. Find the co-ordinates of the points of intersection.

1 (c) The y-axis is a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

(i) Prove that $f^2 = c$.

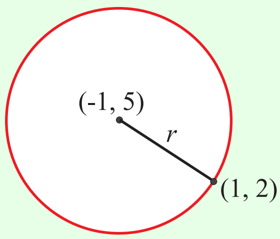
(ii) Find the equations of the circles that pass through the points $(-3, 6)$ and $(-6, 3)$ and have the y-axis as a tangent.

SOLUTION

1 (a)

Circle C with centre (h, k), radius r.

$(x-h)^2 + (y-k)^2 = r^2$ 2



Centre $(-1, 5)$, $r = \sqrt{(-1-1)^2 + (5-2)^2} = \sqrt{4+9} = \sqrt{13}$

Circle: $(x+1)^2 + (y-5)^2 = 13$

Multiplying this equation also gives $x^2 + y^2 + 2x - 10y + 13 = 0 = 13$

1 (b) (i)

If a point is on the circle you can substitute it into the circle equation.

$\therefore 25 + 4 + 5p - 4 + 5 = 0 \Rightarrow 5p = -30 \Rightarrow p = -6$

1 (b) (ii)

STEPS
1. Isolate x or y using equation of the line.
2. Substitute into the equation of the circle and solve simultaneously.

1. L: $x - y - 1 = 0 \Rightarrow x = y + 1$

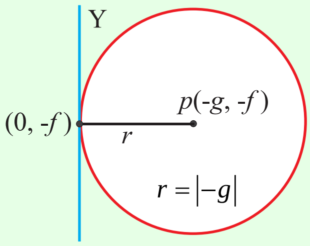
2. K: $x^2 + y^2 - 6x - 2y + 5 = 0 \Rightarrow (y+1)^2 + y^2 - 6(y+1) - 2y + 5 = 0$

$\Rightarrow y^2 + 2y + 1 + y^2 - 6y - 6 - 2y + 5 = 0 \Rightarrow 2y^2 - 6y = 0$

$\Rightarrow y^2 - 3y = 0 \Rightarrow y(y-3) = 0 \Rightarrow y = 0, 3 \Rightarrow x = 1, 4$

Ans: Points of intersection are (1, 0) and (4, 3).

1 (c) (i)



$\Rightarrow r^2 = g^2 = g^2 + f^2 - c$

$\Rightarrow \boxed{c = f^2}$

1 (c) (ii)

$(-3,\ 6)$ is on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$

$\Rightarrow 9 + 36 - 6g + 12f + c = 0 \Rightarrow 6g - 12f - c = 45 ... (\mathbf{1})$

$(-6,\ 3)$ is on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$

$\Rightarrow 36 + 9 - 12g + 6f + c = 0 \Rightarrow 12g - 6f - c = 45 ... (\mathbf{2})$

Y-axis is a tangent $\Rightarrow c = f^2 ... (\mathbf{3})$

Now look at the three equations. Eliminate g from equations (**1**) and (**2**).

$$\begin{array}{l} 12g - 6f - c = 45 ... (\mathbf{2}) \\ 6g - 12f - c = 45 ... (\mathbf{1}) \times (-2) \end{array} \rightarrow \begin{array}{r} 12g - 6f - c = 45 \\ -12g + 24f + 2c = -90 \\ \hline 18f + c = -45 .... (\mathbf{4}) \end{array}$$

Substitute equation **3** into **4**.

$18f + c = -45 \Rightarrow 18f + f^2 = -45 \Rightarrow f^2 + 18f + 45 = 0$

$\Rightarrow (f + 15)(f + 3) = 0 \Rightarrow f = -15, -3$

Using equation **3**: $c = f^2 \Rightarrow c = 225,\ 9$

Using equation **1**: $6g - 12f - c = 45 \Rightarrow g = \dfrac{45 + 12f + c}{6} \Rightarrow g = 15,\ 3$

Therefore the two equations are:

$x^2 + y^2 + 6x - 6y + 9 = 0$ and $x^2 + y^2 + 30x - 30y + 225 = 0$.