CIRCLE (Q 1, PAPER 2)

2001

1 (a) A circle with centre $(-3, 7)$ passes through the point $(5, -8)$. Find the equation of the circle.

1 (b) The equation of a circle is $(x+1)^2+(y-8)^2=160$. The line $x-3y+25=0$ intersects the circle at the points p and q.

(i) Find the co-ordinates of p and the co-ordinates of q.

(ii) Investigate if $[pq]$ is a diameter of the circle.

1 (c) The circle $x^2+y^2+2gx+2fy+c=0$ passes through the points (3, 3) and (4, 1). The line $3x-y-6=0$ is a tangent to the circle at (3, 3).

(i) Find the real numbers g, f and c.

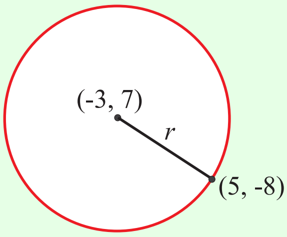
(ii) Find the co-ordinates of the point on the circle at which the tangent parallel to $3x-y-6=0$ touches the circle.

SOLUTION

1 (a)

Circle C with centre (h, k), radius r.

$(x-h)^2+(y-k)^2=r^2$ **2**



$r=\sqrt{(-3-5)^2+(7+8)^2}=\sqrt{64+225}=\sqrt{289}$

Equation of circle: $(x+3)^2+(y-7)^2=289$

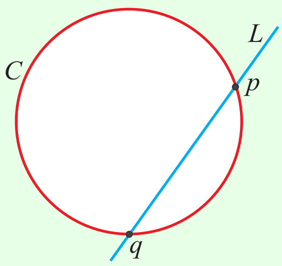
This answer is fine, but you can multiply it out to get: $x^2+y^2+6x-14y-231=0$

1 (b) (i)

The line L intersects the circle C at two points, p and q. To find p and q follow the steps.

STEPS

1. Isolate x or y using equation of the line.
2. Substitute into the equation of the circle and solve simultaneously.



L: $x-3y+25=0 \Rightarrow x=3y-25$

C: $(x+1)^2+(y-8)^2=160 \Rightarrow (3y-25+1)^2+(y-8)^2=160$

$\Rightarrow (3y-24)^2+(y-8)^2=160 \Rightarrow (3[y-8])^2+(y-8)^2=160$

$\Rightarrow 9(y-8)^2+(y-8)^2=160 \Rightarrow 10(y-8)^2=160$

$\Rightarrow (y-8)^2=16 \Rightarrow y-8=\pm 4 \Rightarrow y=4, 12 \Rightarrow x=-13, 11$

Ans: $p(-13, 4)$, $q(11, 12)$

1 (b) (ii)

[*pq*] is a diameter if the midpoint of [*pq*] is the centre of the circle

OR

the length of [*pq*] equals the diameter (twice the radius) of the circle.

Midpoint of [*pq*]: $\left(\frac{-13+11}{2}, \frac{4+12}{2}\right) = (-1, 8)$

Centre of circle: $(-1, 8)$

Therefore, [*pq*] is the diameter.

OR

$|pq| = \sqrt{(-13-11)^2+(4-12)^2} = \sqrt{576+64} = \sqrt{640} = 8\sqrt{10}$

Radius of circle $r = \sqrt{160} = 4\sqrt{10} \Rightarrow 2r = 8\sqrt{10}$

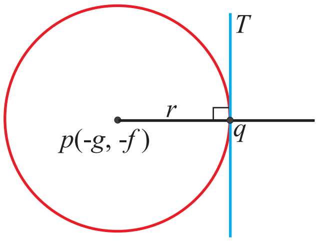
Therefore, [*pq*] is the diameter.

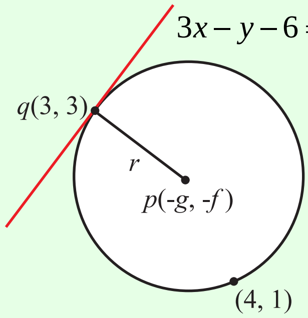
1 (c) (i)

FIND THE EQUATION OF A CIRCLE GIVEN A TANGENT.

SOME POINTS TO NOTE:

1. The tangent *T* is perpendicular to the line *pq*.
2. The perpendicular distance of the centre to the tangent *T* equals the radius *r*.
3. Distance $|pq| = r$.





Equation **1**: (3, 3) is on the circle so you can substitute it in.

$x^2+y^2+2gx+2fy+c=0 \Rightarrow 9+9+6g+6f+c=0$

$\Rightarrow 6g+6f+c=-18.....(\mathbf{1})$

Equation **2**: (4, 1) is on the circle so you can substitute it in.

$x^2+y^2+2gx+2fy+c=0 \Rightarrow 16+1+8g+2f+c=0$

$\Rightarrow 8g+2f+c=-17.....(\mathbf{2})$

Equation **3**: Use point No. 1 above.

Slope of $T = 3 \Rightarrow$ Slope of $pq = -\frac{1}{3}$

Slope of $pq = \frac{3+f}{3+g} = -\frac{1}{3} \Rightarrow 9+3f = -3-g \Rightarrow g+3f = -12....(\mathbf{3})$

Eliminate *c* from equations **1** and **2**:

$$\begin{aligned} 6g+6f+c &= -18...(\mathbf{1})\times(-1) \\ 8g+2f+c &= -17...(\mathbf{2}) \end{aligned} \quad \rightarrow \quad \begin{aligned} -6g-6f-c &= +18 \\ 8g+2f+c &= -17 \\ \hline 2g-4f &= 1....(\mathbf{4}) \end{aligned}$$

Eliminate g from equations **3** and **4**.

$$\begin{aligned} g+3f &= -12 \ldots(\mathbf{3})\times(-2) \\ 2g-4f &= 1 \ldots(\mathbf{4}) \end{aligned} \quad \to \quad \begin{aligned} -2g-6f &= 24 \\ 2g-4f &= 1 \\ \hline -10f &= 25 \Rightarrow f = -\tfrac{5}{2} \end{aligned}$$

Substitute this value of f into equation **3** to find g:

$$g+3f=-12 \Rightarrow g+3(-\tfrac{5}{2})=-12 \Rightarrow g=-12+\tfrac{15}{2}=-\tfrac{9}{2}$$

Substitute these values of g and f into equation **1** to find c:

$$6g+6f+c=-18 \Rightarrow 6(-\tfrac{9}{2})+6(-\tfrac{5}{2})+c=-18$$

$$\Rightarrow -27-15+c=-18 \Rightarrow c=24$$

Ans: $g=-\tfrac{9}{2},\ f=-\tfrac{5}{2},\ c=24$

1 (c) (ii)

The point q is translated through p and on to r.

$(3, 3) \to (\tfrac{9}{2}, \tfrac{5}{2}) \to (6, 2)$

Ans: (6, 2)