CIRCLE (Q 1, PAPER 2)

1996

1 (a) The parametric equations of a circle are

$x = 5 + \frac{\sqrt{3}}{2}\cos\theta,\ y = -3 + \frac{\sqrt{3}}{2}\sin\theta.$

Find its Cartesian equation.

(b) Points $(1, -1)$, $(-6, -2)$ and $(3, -5)$ are on a circle C.
Find the equation of C.

(c) S_1: $x^2 + y^2 - 6x - 4y + 12 = 0$

S_2: $x^2 + y^2 + 10x + 4y + 20 = 0$ are two circles.

(i) Find the coordinates of their centres p and q and the lengths of their radii r_1, r_2 respectively.

(ii) Verify that the lines
L: $y - 1 = 0$ and M: $4x - 3y - 1 = 0$
are tangents to S_1.

(iii) If w is the point of intersection of L and M and $w \in [pq]$, show that

$|pw| : |wq| = r_1 : r_2.$

ANSWERS

1 (a) $4(x-5)^2 + 4(y+3)^2 = 3$

(b) $x^2 + y^2 + 4x + 10y + 4 = 0$

(c) (i) $p(3, 2)$, $r_1 = 1$; $q(-5, -2)$, $r_2 = 3$