ALGEBRA (Q 1 & 2, PAPER 1)

SOLUTIONS NO. 5: CUBIC EQUATIONS

2005

2 (b) The cubic equation $4x^3+10x^2-7x-3=0$ has one integer root and two irrational roots. Express the irrational roots in simplest surd form.

SOLUTION

The integer root must be a factor of the last term of the cubic expression. It can be any of the following numbers: $-3, -1, 1, 3$.

$f(-3)=4(-3)^3+10(-3)^2-7(-3)-3=-108+90+21-3=0$

Therefore, -3 is a root $\Rightarrow (x+3)$ is a factor.

If you divide $(x + 3)$ into the cubic, you will get the quadratic factor. You can also find the quadratic factor by the lining up process.

METHOD 1: DIVISION

$$
\begin{array}{r|l}
 & 4x^2-2x-1 \\
x+3 & 4x^3+10x^2-7x-3 \\
 & \mp 4x^3 \mp 12x^2 \\
\hline
 & \quad -2x^2-7x-3 \\
 & \quad \pm 2x^2 \pm 6x \\
\hline
 & \qquad -x-3 \\
 & \qquad \pm x \pm 3 \\
\hline
 & \qquad\quad 0
\end{array}
$$

METHOD 2: LINING UP

A cubic is a linear multiplied by a quadtatic. You can find the first term and the last term of the quadratic as the first term by the first term gives the first term and the last by the last gives the last. The middle term in the quadratic is unknown so call it ax.

$4x^3+10x^2-7x-3=(x+3)(4x^2+ax-1)$

$\Rightarrow 4x^3+10x^2-7x-3=4x^3+(a+12)x^2+(3a-1)x-3$

Lining up: $a+12=10 \Rightarrow a=-2$

Therefore, the quadratic factor is $4x^2-2x-1$.

Therefore, $4x^3+10x^2-7x-3=(x+3)(4x^2-2x-1)=0$

Solve the quadratic using the formula.

$a=4,\ b=-2,\ c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{2\pm\sqrt{4+16}}{8}=\frac{2\pm\sqrt{20}}{8}=\frac{2\pm2\sqrt{5}}{8}=\frac{1\pm\sqrt{5}}{4}$$

2002

1 (b) The cubic equation $x^3-4x^2+9x-10=0$ has one integer root and two complex roots. Find the three roots.

SOLUTION

The integer root must be a factor of the last term of the cubic expression. It can be any of the following numbers: $-10, -5, -1, 1, 5, 10$.

$f(2)=(2)^3-4(2)^2+9(2)-10=8-16+18-10=0$

Therefore, 2 is a root $\Rightarrow (x-2)$ is a factor.

If you divide $(x-2)$ into the cubic, you will get the quadratic factor. You can also find the quadratic factor by the lining up process.

METHOD 1: DIVISION

$$
\begin{array}{r|l}
 & x^2-2x+5 \\ \hline
(x-2) & x^3-4x^2+9x-10 \\
 & \mp x^3 \pm 2x^2 \\ \hline
 & -2x^2+9x-10 \\
 & \pm 2x^2 \mp 4x \\ \hline
 & 5x-10 \\
 & \mp 5x \pm 10 \\ \hline
 & 0
\end{array}
$$

METHOD 2: LINING UP

A cubic is a linear multiplied by a quadtatic. You can find the first term and the last term of the quadratic as the first term by the first term gives the first term and the last by the last gives the last. The middle term in the quadratic is unknown so call it *ax*.

$x^3-4x^2+9x-10=(x-2)(x^2+ax+5)$

$\Rightarrow x^3-4x^2+9x-10=x^3+(a-2)x^2+(5-2a)x-10$

Lining up: $a-2=-4 \Rightarrow a=-2$

Therefore, the quadratic factor is x^2-2x+5.

$\therefore x^3-4x^2+9x-10=(x-2)(x^2-2x+5)$

Solve the quadratic using the formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{2\pm\sqrt{4-20}}{2}=\frac{2\pm\sqrt{-16}}{2}=\frac{2\pm 4i}{2}=1\pm 2i$$

ANSWER: $2,\ 1\pm 2i$