ALGEBRA (Q 1 & 2, PAPER 1)

SOLUTIONS NO. 3: FACTOR THEOREM

2006

1 (b) $f(x) = 3x^3 + mx^2 - 17x + n$, where m and n are constants. Given that $x-3$ and $x+2$ are factors of $f(x)$, find the value of m and the value of n.

SOLUTION

$x-3$ is a factor $\Rightarrow f(3) = 3(3)^3 + m(3)^2 - 17(3) + n = 0 \Rightarrow 9m + n = -30$.....(**1**)

$x+2$ is a factor $\Rightarrow f(-2) = 3(-2)^3 + m(-2)^2 - 17(-2) + n = 0 \Rightarrow 4m + n = -10$.....(**2**)

Solving equation **1** and **2** simultaneously: $m = -4,\ n = 6$

2004

1 (b) (i) Let $f(x) = x^3 + kx^2 - 4x - 12$, where k is a constant. Given that $x+3$ is a factor of $f(x)$, find the value of k.

SOLUTION

If $x+3$ is a factor $\Rightarrow f(-3) = (-3)^3 + k(-3)^2 - 4(-3) - 12 = 0$

$\Rightarrow -27 + 9k + 12 - 12 = 0 \Rightarrow 9k = 27 \Rightarrow k = 3$

2003

1 (b) (i) $f(x) = ax^2 + bx + c$ where $a, b, c \in \mathbf{R}$. Given that k is a real number such that $f(k) = 0$, prove that $x-k$ is a factor of $f(x)$.

(ii) Show that $2x - \sqrt{3}$ is a factor of $4x^2 - 2(1+\sqrt{3})x + \sqrt{3}$ and find the other factor.

SOLUTION

1 (b) (i)

The proof of the factor theorem for a cubic is given in the book. You are asked to prove it for a quadratic, which is easier.

PROOF OF FACTOR THEOREM

$f(x) = ax^2 + bx + c$

$f(k) = ak^2 + bk + c$

$\therefore f(x) - f(k) = a(x^2 - k^2) + b(x-k)$

$= a(x+k)(x-k) + b(x-k)$

$= (x-k)\{a(x+k) + b\} = (x-k)g(x)$

$\therefore f(x) = f(k) + (x-k)g(x)$

(i) $f(k) = 0 \Rightarrow f(x) = (x-k)g(x) \therefore x-k$ is a factor.

(ii) $x-k$ is a factor $\Rightarrow f(k) = 0$.

1 (b) (ii)

If $2x-\sqrt{3}$ is a factor of $f(x) \Rightarrow f(\frac{\sqrt{3}}{2}) = 0$

$\therefore 4(\frac{\sqrt{3}}{2})^2 - 2(1+\sqrt{3})(\frac{\sqrt{3}}{2}) + \sqrt{3} = 3 - \sqrt{3} - 3 + \sqrt{3} = 0$

Therefore, $2x-\sqrt{3}$ is a factor.
You can find the other factor by division or by lining up. Lining up is better here, I think, but you choose whichever option you prefer.
A quadraticis a linear multiplied by a linear. The first term in the quadratic equals the first term in the linear multiplied by the first term in the linear. Also, the last term in the quadratic equals the last term in the linear multiplied by the last term in the linear.

$\therefore 4x^2 - 2(1+\sqrt{3})x + \sqrt{3} = (2x-\sqrt{3})(2x-1)$

$\therefore (2x-1)$ is the other factor.

2001

1 (b) Let $f(x) = 2x^3 + mx^2 + nx + 2$ where m and n are constants. Given that $x-1$ and $x+2$ are factors of $f(x)$, find the value of m and the value of n.

SOLUTION

$x-1$ is a factor $\Rightarrow f(1) = 2(1)^3 + m(1)^2 + n(1) + 2 = 0 \Rightarrow m + n = -4$.....(**1**)

$x+2$ is a factor $\Rightarrow f(-2) = 2(-2)^3 + m(-2)^2 + n(-2) + 2 = 0 \Rightarrow 2m - n = 7$.....(**2**)

Solving equation **1** and **2** simultaneously: $m = 1,\ n = -5$