ALGEBRA (Q 1 & 2, PAPER 1)

LESSON NO. 4: QUADRATIC EQUATIONS

2006

2 (b) (i) Find the range of values of $t \in \mathbf{R}$ for which the quadratic equation $(2t-1)x^2 + 5tx + 2t = 0$ has real roots.

(ii) Explain why the roots are real when t is an integer.

2004

2 (b) (ii) The roots of $x^2 + px + q = 0$ are α and β, where $p, q \in \mathbf{R}$. Find the quadratic equation whose roots are $\alpha^2\beta$ and $\alpha\beta^2$.

2 (c) (ii) Show that for any real values of a, b and h, the quadratic equation $(x-a)(x-b) - h^2 = 0$ has real roots.

2003

1 (c) The real roots of $x^2 + 10x + c = 0$ differ by $2p$ where $c,\ p \in \mathbf{R}$ and $p > 0$.

(i) Show that $p^2 = 25 - c$.

(ii) Given that one root is greater than zero and the other root is less than zero, find the range of possible values of p.

2 (c) (ii) Given that $x = \alpha$ and $x = \beta$ are the solutions of the quadratic equation $2k^2x^2 + 2ktx + t^2 - 3k^2 = 0$ where $k, t \in \mathbf{R}$ and $k \neq 0$, show that $\alpha^2 + \beta^2$ is independent of k and t.

2002

1 (c) $(p+r-t)x^2 + 2rx + (t+r-p) = 0$ is a quadratic equation, where p, r, and t are integers. Show that

(i) the roots are rational
(ii) one of the roots is an integer.

2 (c) (i) Show that if the roots of $x^2 + bx + c = 0$ differ by 1, then $b^2 - 4c = 1$.

(ii) The roots of the equation $x^2 + (4k-5)x + k = 0$ are consecutive integers. Using the result from part (**i**), or otherwise, find the value of k and the roots of the equation.

2001

2 (c) α and β are real numbers such that $\alpha + \beta = -7$ and $\alpha\beta = 11$.

(i) Show that $\alpha^2 + \beta^2 = 27$.

(ii) Find a quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ and write your answer in the form

$px^2 + qx + r = 0$ where $p, q, r \in \mathbf{Z}$.

ANSWERS

2006 2 (b) (i) $t \le -\frac{8}{9}$ and $t \ge 0$

(ii) The roots are unreal for the range of values $-\frac{8}{9} \le t \le 0$. There are no integers (whole numbers) in this range.

2004 2 (b) (ii) $x^2 + pqx + q^3 = 0$

2003 1 (c) (ii) $p > 5$

2002 2 (c) (ii) $k = 2$; Roots: $-2, -1$

2001 2 (c) (ii) $11x^2 - 27x + 11 = 0$