ALGEBRA (Q 1 & 2, PAPER 1)

2011

1 **(a)** Simplify fully $\frac{x+1}{x-1}-\frac{x-1}{x+1}-\frac{4}{x^2-1}$.

(b) **(i)** Prove the factor theorem for polynomials of degree 2.

That is, given that $f(x)=ax^2+bx+c$ and k is a number such that $f(k)=0$, prove that $(x-k)$ is a factor of $f(x)$.

(ii) The factor theorem also holds for polynomials of higher degree.
Find the values of n for which $(x+k)$ is a factor of the polynomial
$g(x)=x^n+k^n$, where $k\neq 0$.

(c) $(x-a)^2$ is a factor of $2x^3-5ax^2+8abx-36a$, where $a\neq 0$.
Find the possible values of a and b.

2. **(a)** Solve for x: $|2x-1|\leq 3$, where $x\in\mathbb{R}$.

(b) α and $\frac{1}{\alpha}$ are the roots of the quadratic equation $3kx^2-18tx+(2k+3)=0$,
where t and k are constants.

(i) Find the value of k.

(ii) If one of the roots is four times the other, find the possible values of t.

(c) Let $f(x)=\frac{1}{x^2-6x+a}$, where a is a constant.

(i) Prove that if $a = 13$, then $f(x) > 0$ for all $x\in\mathbb{R}$.

(ii) Prove that if $a = 13$, then $f(x) < 1$ for all $x\in\mathbb{R}$.

(iii) Find the range of values of a such that $0 < f(x) < 1$, for all $x\in\mathbb{R}$.

ANSWERS

1 (a) $\frac{4}{x+1}$
(b) (ii) $n = \{3, 5, 7, ...\}$
(c) $a = \pm 6,\ b = \pm 3$

2 (a) $-1\leq x\leq 2$
(b) (i) $k = 3$ (ii) $t = \pm\frac{5}{4}$
(c) (iii) $a > 10$