ALGEBRA (Q 1 & 2, PAPER 1)

2010

1 (a) $x^2 - 6x + t = (x+k)^2$, where t and k are constants.
Find the value of k and the value of t.

(b) Given that p is a real number, prove that the equation $x^2 - 4px - x + 2p = 0$ has real roots.

(c) $(x - 2)$ and $(x + 1)$ are factors of $x^3 + bx^2 + cx + d$.

(i) Express c in terms of b.

(ii) Express d in terms of b.

(iii) Given that b, c and d are three consecutive terms in an arithmetic sequence, find their values.

2. (a) Solve the simultaneous equations

$$2x + 3y = 0$$
$$x + y + z = 0$$
$$3x + 2y - 4z = 9.$$

(b) The equation $x^2 - 12x + 16 = 0$ has roots α^2 and β^2, where $\alpha > 0$ and $\beta > 0$.

(i) Find the value of $\alpha\beta$.

(ii) Hence, find the value of $\alpha + \beta$.

(c) (i) Prove that for all real numbers a and b,

$$a^2 - ab + b^2 \geq ab.$$

(ii) Let a and b be non-zero real numbers such that $a + b \geq 0$.

Show that $\dfrac{a}{b^2} + \dfrac{b}{a^2} \geq \dfrac{1}{a} + \dfrac{1}{b}$.

ANSWERS

1 (a) $k = -3$, $t = 9$
(c) (i) $c = -b - 3$ (ii) $d = -2b - 2$ (iii) $-4, 1, 6$

2 (a) $x = 3$, $y = -2$, $z = -1$
(b) (i) $\alpha\beta = 4$ (ii) $\alpha + \beta = 2\sqrt{5}$