

ALGEBRA (Q 1 & 2, PAPER 1)

2007

1 (a) Simplify $\frac{x^2 - xy}{x^2 - y^2}$.

(b) Let $f(x) = x^2 + (k+1)x - k - 2$, where k is a constant.

(i) Find the value of k for which $f(x) = 0$ has equal roots.

(ii) Find, in terms of k , the roots $f(x) = 0$.

(iii) Find the range of values of k for which both roots are positive.

(c) $x + p$ is a factor of both $ax^2 + b$ and $ax^2 + bx - ac$.

(i) Show that $p^2 = -\frac{b}{a}$ and that $p = \frac{-b - ac}{b}$.

(ii) Hence show that $p^2 + p^3 = c$.

2. (a) Solve the simultaneous equations

$$x + y + z = 2$$

$$2x + y + z = 3$$

$$x - 2y + 2z = 15$$

(b) α and β are the roots of the equation $x^2 - 4x + 6 = 0$.

(i) Find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

(ii) Find the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(c) (i) Prove that $x + \frac{9}{x+2} \geq 4$, where $x + 2 > 0$.

(ii) Prove that $x + \frac{9}{x+a} \geq 6 - a$, where $x + a > 0$.

ANSWERS

1 (a) $\frac{x}{x+y}$

(b) (i) $k = -3$ (ii) $1, -k - 2$ (iii) $k < -2$

2 (a) $x = 1, y = -3, z = 4$

(b) (i) $\frac{2}{3}$ (ii) $6x^2 - 4x + 1 = 0$