

ALGEBRA (Q 1 & 2, PAPER 1)

2006

1 (a) Find the real number a such that for all $x \neq 9$, $\frac{x-9}{\sqrt{x}-3} = \sqrt{x} + a$.

1 (b) $f(x) = 3x^3 + mx^2 - 17x + n$, where m and n are constants. Given that $x-3$ and $x+2$ are factors of $f(x)$, find the value of m and the value of n .

1 (c) $x^2 - t$ is a factor of $x^3 - px^2 - qx + r$.

(i) Show that $pq = r$.

(ii) Express the roots of $x^3 - px^2 - qx + r = 0$ in terms of p and q .

2 (a) Solve the simultaneous equations:

$$y = 2x - 5$$

$$x^2 + xy = 2$$

2 (b) (i) Find the range of values of $t \in \mathbf{R}$ for which the quadratic equation

$$(2t-1)x^2 + 5tx + 2t = 0$$
 has real roots.

(ii) Explain why the roots are real when t is an integer.

2 (c) $f(x) = 1 - b^{2x}$ and $g(x) = b^{1+2x}$, where b is a positive real number. Find, in terms of b , the value of x for which $f(x) = g(x)$.

ANSWERS

1 (a) $a = 3$

1 (b) $m = -4$, $n = 6$

1 (c) $x = p, \pm\sqrt{q}$

2 (a) $x = -\frac{1}{3}, 2$; $y = -\frac{17}{3}, -1$

2 (b) (i) $t \leq -\frac{8}{9}$ and $t \geq 0$

(ii) The roots are unreal for the range of values $-\frac{8}{9} \leq t \leq 0$. There are no integers (whole numbers) in this range.

2 (c) $-\frac{\log_{10}(1+b)}{2\log_{10} b}$ or $-\log_b \sqrt{b+1}$