ALGEBRA (Q 1 & 2, PAPER 1)

2004

1 (a) Express $\frac{1-\sqrt{3}}{1+\sqrt{3}}$ in the form $a\sqrt{3}-b$, where a and $b \in \mathbf{N}$.

(b) (i) Let $f(x)=x^3+kx^2-4x-12$, where k is a constant. Given that $x+3$ is a factor of $f(x)$, find the value of k.

(ii) Show that $\frac{3}{1+x^p}+\frac{3}{1+x^{-p}}$ simplifies to a constant.

(c) (i) Show that $p^3+q^3-(p+q)^3=-3pq(p+q)$.

(ii) Hence, or otherwise, find, in terms of a and b, the three values of x for which $(a-x)^3+(b-x)^3-(a+b-2x)^3=0$.

2 (a) Solve, without using a calculator, the following simultaneous equations:

$$\begin{aligned} 3x+y+z&=0\\ x-y+z&=2\\ 2x-3y-z&=9 \end{aligned}$$

(b) (i) Solve the inequality $\frac{x+1}{x-1}<4$, where $x \in \mathbf{R}$ and $x \neq 1$.

(ii) The roots of $x^2+px+q=0$ are α and β, where $p, q \in \mathbf{R}$. Find the quadratic equation whose roots are $\alpha^2\beta$ and $\alpha\beta^2$.

(c) (i) $f(x)=2x+1$, for $x \in \mathbf{R}$. Show that there exists a real number k such that for all x, $f(x+f(x))=kf(x)$.

(ii) Show that for any real values of a, b and h, the quadratic equation $(x-a)(x-b)-h^2=0$ has real roots.

ANSWERS

1 (a) $\sqrt{3}-2$

1 (b) (i) $k=3$

1 (c) (ii) $a,\ b,\ \frac{1}{2}(a+b)$

2 (a) $x=1,\ y=-2,\ z=-1$

2 (b) (i) $x<1,\ x>\frac{5}{3}$

2 (b) (ii) $x^2+pqx+q^3=0$

2 (c) (i) $k=3$