ALGEBRA (Q 1 & 2, PAPER 1)

2003

1 (a) Express the following as a single fraction in its simplest form: $\dfrac{6y}{x(x+4y)}-\dfrac{3}{2x}$.

(b) (i) $f(x)=ax^2+bx+c$ where $a, b, c \in \mathbf{R}$. Given that k is a real number such that $f(k)=0$, prove that $x-k$ is a factor of $f(x)$.

(ii) Show that $2x-\sqrt{3}$ is a factor of $4x^2-2(1+\sqrt{3})x+\sqrt{3}$ and find the other factor.

(c) The real roots of $x^2+10x+c=0$ differ by $2p$ where $c, p \in \mathbf{R}$ and $p > 0$.

(i) Show that $p^2=25-c$.

(ii) Given that one root is greater than zero and the other root is less than zero, find the range of possible values of p.

2 (a) Solve the simultaneous equations:

$$3x-y=8$$
$$x^2+y^2=10$$

(b) (i) Solve for x: $|4x+7|<1$.

(ii) Given that x^2-ax-3 is a factor of x^3-5x^2+bx+9 where $a, b \in \mathbf{R}$, find the value of a and the value of b.

(c) (i) Solve for y: $2^{2y+1}-5(2^y)+2=0$.

(ii) Given that $x=\alpha$ and $x=\beta$ are the solutions of the quadratic equation $2k^2x^2+2ktx+t^2-3k^2=0$ where $k, t \in \mathbf{R}$ and $k \neq 0$, show that $\alpha^2+\beta^2$ is independent of k and t.

ANSWERS

1 (a) $-\dfrac{3}{2(x+4y)}$

1 (b) (ii) $(2x-1)$

1 (c) (ii) $p > 5$

2 (a) $x=3, \frac{9}{5}$; $y=1, -\frac{13}{5}$

2 (b) (i) $-2<x<-\frac{3}{2}$

2 (b) (ii) $a = 2$, $b = 3$

2 (c) (i) $y=-1, 1$