

ALGEBRA (Q 1 & 2, PAPER 1)

2002

1 (a) Solve the equation $x = \sqrt{x+2}$.

(b) The cubic equation $x^3 - 4x^2 + 9x - 10 = 0$ has one integer root and two complex roots. Find the three roots.

(c) $(p+r-t)x^2 + 2rx + (t+r-p) = 0$ is a quadratic equation, where p , r , and t are integers. Show that

(i) the roots are rational

(ii) one of the roots is an integer.

2 (a) Solve, without using a calculator, the following simultaneous equations:

$$x + 2y + 4z = 7$$

$$x + 3y + 2z = 1$$

$$-y + 3z = 8$$

(b) (i) Find the range of values of $x \in \mathbf{R}$ for which $x^2 + x - 20 \leq 0$.

(ii) Let $g(x) = x^n + 3$, for all $x \in \mathbf{R}$, where $n \in \mathbf{N}$. Show that if n is odd then $g(x) + g(-x)$ is constant.

(c) (i) Show that if the roots of $x^2 + bx + c = 0$ differ by 1, then $b^2 - 4c = 1$.

(ii) The roots of the equation $x^2 + (4k - 5)x + k = 0$ are consecutive integers. Using the result from part (i), or otherwise, find the value of k and the roots of the equation.

ANSWERS

1 (a) $x = 2$

1 (b) $2, 1 \pm 2i$

2 (a) $x = 3, y = -2, z = 2$

2 (b) (i) $-5 \leq x \leq 4$

2 (c) (ii) $k = 2$; Roots: $-2, -1$