ALGEBRA (Q 1 & 2, PAPER 1)

2001

1 (a) Find the real numbers a and b such that $x^2 + 4x - 6 = (x + a)^2 + b$ for all $x \in \mathbf{R}$.

(b) Let $f(x) = 2x^3 + mx^2 + nx + 2$ where m and n are constants. Given that $x - 1$ and $x + 2$ are factors of $f(x)$, find the value of m and the value of n.

(c) $x^2 - px + q$ is a factor of $x^3 + 3px^2 + 3qx + r$.

(i) Show that $q = -2p^2$.

(ii) Show that $r = -8p^3$.

(iii) Find the three roots of $x^3 + 3px^2 + 3qx + r = 0$ in terms of p.

2 (a) Solve the simultaneous equations:

$$x - y = 0$$
$$(x+2)^2 + y^2 = 10$$

(b) (i) Solve for x: $|3x + 5| < 4$.

(ii) Simplify $\left(x^2 + \sqrt{2} + \frac{1}{x^2}\right)\left(x^2 - \sqrt{2} + \frac{1}{x^2}\right)$ and express your answer in the form $x^n + \frac{1}{x^n}$ where n is a whole number.

(c) α and β are real numbers such that $\alpha + \beta = -7$ and $\alpha\beta = 11$.

(i) Show that $\alpha^2 + \beta^2 = 27$.

(ii) Find a quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ and write your answer in the form $px^2 + qx + r = 0$ where $p, q, r \in \mathbf{Z}$.

ANSWERS

1 (a) $a = 2, b = -10$

1 (b) $m = 1, n = -5$

1 (c) $x = -4p, -p, 2p$

2 (a) $x = -3, 1;\ y = -3, 1$

2 (b) (i) $-3 < x < -\frac{1}{3}$ (ii) $x^4 + \frac{1}{x^4}$

2 (c) (ii) $11x^2 - 27x + 11 = 0$