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ALGEBRA (Q 1 & 2, PAPER 1)

2000

1 (a) Show that the following simplifies to a constant when $x \neq 2$

$$\frac{3x-5}{x-2} + \frac{1}{2-x}.$$

(b) $f(x) = ax^3 + bx^2 + cx + d$ where $a, b, c, d \in \mathbf{R}$.

If k is a real number such that $f(k) = 0$, prove that $x - k$ is a factor of $f(x)$.

(c) $(x-t)^2$ is a factor of $x^3 + 3px + c$.

Show that

(i) $p = -t^2$

(ii) $c = 2t^3$.

2 (a) Solve for x, y, z

$$\begin{aligned} 3x - y + 3z &= 1 \\ x + 2y - 2z &= -1 \\ 4x - y + 5z &= 4 \end{aligned}$$

(b) Solve $x^2 - 2x - 24 = 0$.

Hence, find the values of x for which

$$\left(x + \frac{4}{x}\right)^2 - 2\left(x + \frac{4}{x}\right) - 24 = 0, \; x \in \mathbf{R}, \; x \neq 0.$$

(c) (i) Express $a^4 - b^4$ as a product of three factors.

(ii) Factorise $a^5 - a^4b - ab^4 + b^5$.

Use your results from **(i)** and **(ii)** to show that

$a^5 - b^5 > a^4b + ab^4$

where a and b are positive unequal real numbers.

ANSWERS

1 (a) 3

2 (a) $x = -1, \; y = 2, \; z = 2$

2 (b) $x = -4, \; 6; \; x = -2, \; 3 \pm \sqrt{5}$

2 (c) (i) $(a+b)(a-b)(a^2+b^2)$

(ii) $(a+b)(a-b)^2(a^2+b^2)$